LIMPOPO

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GRADE 12

ACTIVITIES MANUAL

2024

TOPIC: ALGEBRA

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TOPIC: ALGEBRA (25±3) MARKS

EXAM GUIDELINE

- Solving quadratic equations by completing the square will NOT be examined.
- Solving quadratic equations using the substitution method (k-method) is examinable.
- Equations involving surds that lead to a quadratic equation are examinable.
- Solution of non-quadratic inequalities should be seen in the context of functions.
- The nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

SUB-TOPIC 1: FACTORISATION

- Factorization is the breaking down of algebraic expressions (binomial and trinomial) into its products.

Example 1 (binomial)	**Solutions**	**Errors and misconceptions**
$x^2+4x=0$	$\mathrm{x}^2+4\mathrm{x}=0$ $\mathrm{x}(\mathrm{x}+4)=0$ $\mathrm{x}=0$ or $\mathrm{x}=-4$	• Most candidates turn to reverse the equation when it come in the form of x(x+4) = 0 while they should apply the zero rule.
Example 2(trinomial in standard form)		
$x^2+7x+12=0$	$x^2+7x+12=0$ $(x+4)(x+3)=0$ $x=-4$ *or* $x=-3$	
Example 3 (trinomial in non-standard form)		
$2x^2-5x=3$ $2x^2-5x-3=0$ $(2x+1)(x-3)=0$ $2x+1=0$ *or* $x-3=0$ $x=-\frac{1}{2}$ *or* $x=3$	Standard form Factorization using trial and error method to solve for x applying the zero rule	• Most candidate don't write this equation first in a correct standard form. • Most candidates apply the zero rule before taking the equation into a standard form.

ACTIVITY 1 (LEVEL 1)

1.1 $x(2-x)=0$ (2)

1.2 $x(x+6)=0$ (2)

1.3 $(x-5)(x+2)=6$ (4)

1.4 $(x-2)(5+x)=0$ (2)

1.5 $x(x+6)+1=0$ (4)

1.6 $x^2-x-20=0$ (2)

1.7 $2x(3-x)=0$ (2)

QUADRATIC EQUATIONS.

- Quadratic equations refer to any expression that can be written in the form of

  $ax^2+bx+c=0$ where a ≠ 0

- The quadratic formulae is given by $\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

Example 1	**Explanation**	**Errors and misconceptions**
$3x^2-5x+1=0$ $a=3 \quad b=-5 \quad c=1$	Standard form Find the value of ***a, b*** and ***c***.	• most candidates fail to identify the values of a, b and c. • wrong substitution • division in the quadratic formula means everything divided by 2a • incorrect rounding off of the decimals
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Use the quadratic formulae.	
$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(1)}}{2(1)}$	Substitute correctly into the formulae.	
$x=\frac{5+\sqrt{13}}{6}$ or $x=\frac{5-\sqrt{13}}{6}$	Simplification	
$x=1.43$ or $x=6.23$	Answer	

ACTIVITY 2 (LEVEL 1)

2.1 $-x^2-2x+1=0$ (correct to TWO decimal places) (3)

2.2 $2x^2-3x-7=0$ (correct to TWO decimal places) (3)

2.3 $3x^2+8x=-2$ (correct to TWO decimal places) (3)

2.4 $3x-4=\frac{2}{x}$ (Hint: kill the fraction first) (correct to TWO decimal places) (4)

2.5 $x^2-9=0$ (2)

2.6 $3^{3x+1}=9^{2x-4}$ (3)

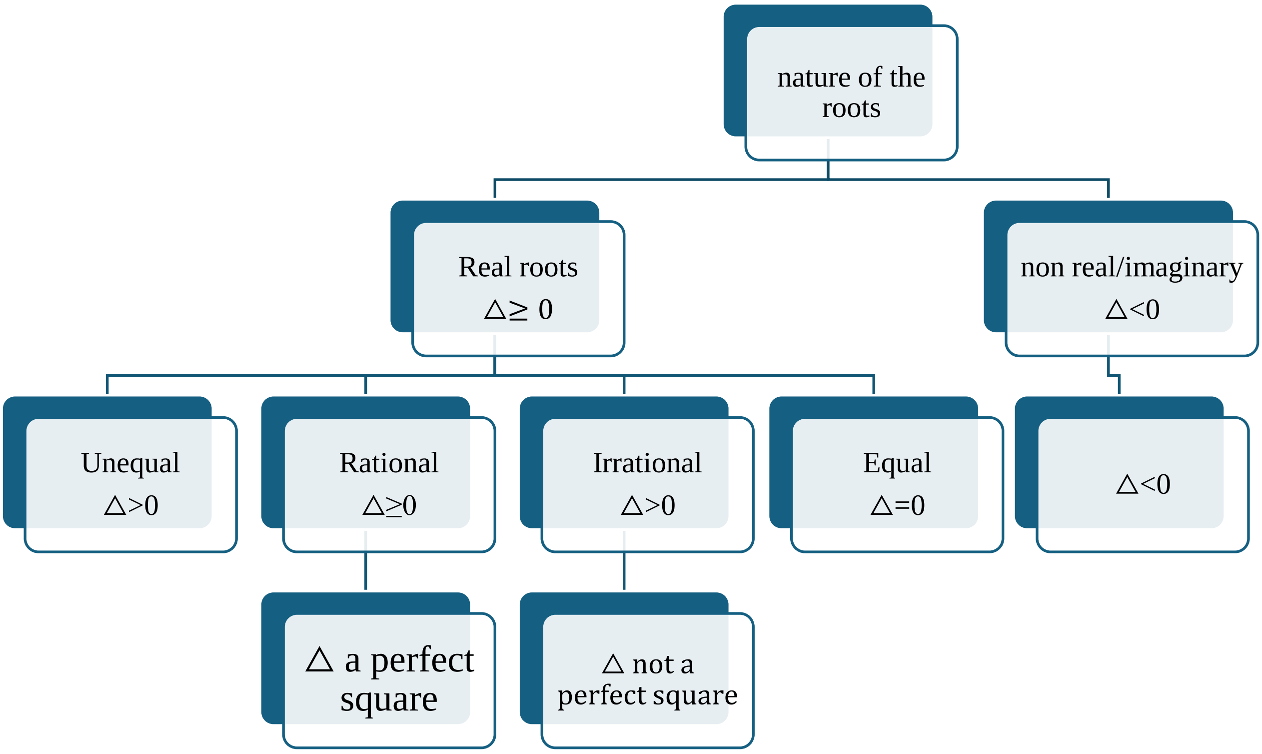
2.7 $x-\frac{3}{X}=-2$ (4)

2.8 $x-5+\frac{2}{x}=0$ (4)

2.9 $x+2=\frac{2}{x+1}$ (4)

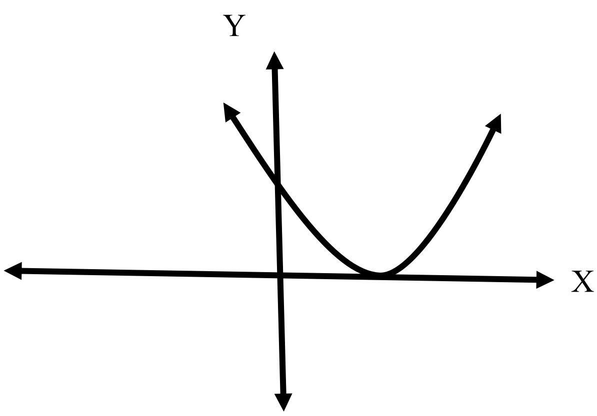
NATURE OF THE ROOTS

The discriminant or delta ($\Delta = b^2 - 4ac$) is used to determine the nature of the roots of quadratic equations. The word 'nature' refers to the types of values that can be either Real (Rational, Irrational), or non-real/imaginary.

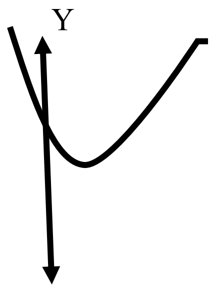


Nature of the roots graphically

Equal roots ($b^2 - 4ac$=0) {The turning point and the x-intercepts of the graph shares the same point}

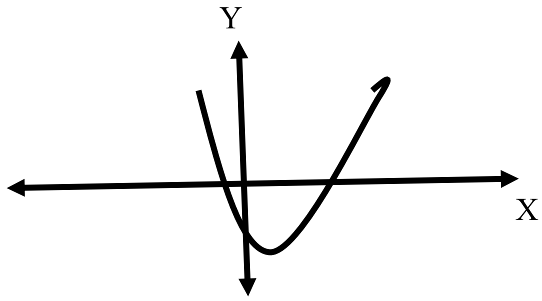


No real roots ($b^2 - 4ac < 0$) {The graph will not cut the x-axis or will not have the x-intercepts}



EXAMPLES	SOLUTIONS	ERRORS
• Show that the roots of $x^2 - 2x - 7 = 0$,are irrational, without solving the equation.	$x^2 - 2x - 7 = 0$ $for\ irrational\ \triangle > 0$ $= b^2 - 4ac$ $= (-2)^2 - 4(1)(-7)$ $= 32 > 0$	
• Show that $x^2 + x + 1 = 0$ $has\ no\ real\ roots$	$x^2 + x + 1 = 0$ $\Delta = b^2 - 4ac$ $= (1)^2 - 4(1)(1)$ $= -3 < 0$ ∴The roots are non-real	
• Show that the roots of $3x^2 + (k+2)x = 1 - k$ $are\ real\ and\ rational\ for\ all\ values\ of\ k$	$3x^2 + (k+2)x = 1 - k$ $3x^2 + (\boldsymbol{k+2})x + \boldsymbol{k-1} = 0$ $\triangle \geq 0$ $b^2 - 4ac \geq 0$ $(k+2)^2 - 4(3)(k-1) \geq 0$ $k^2 + 4k + 4 - 12k + 12 \geq 0$ $k^2 - 8k + 16 \geq 0$ $(k-4)^2 \geq 0$ $k \geq 4$	Write in standard form Identifying the values of **b** and **c** Operation to be used on $\triangle$

Distinct real roots ($b^2 - 4ac > 0$**)** {The graph will cut the x-axis at two different points}



ACTIVITY 3

3.1 The roots of a quadratic equation are $x = \frac{-3 \pm \sqrt{4-p}}{2}$
For which values of p will the roots be real? (2)

3.2 The roots of quadratic equation are given by $x = \frac{-5 \pm \sqrt{20+8k}}{6}$

where k ϵ (-3; -2; -1;0;1;2;3)

3.2.1 write down two values for which the roots will be rational. (2)

3.2.2 write down ONE value of K for which the root will be non- real. (1)

3.3 If 2 is a root of $2x^2 - 3x - p = 0$, determine the value of p and hence the other root. (4)

SOLVE FOR X BY SQUARING BOTH SIDES

EXAMPLE	**STEPS**	**ERROR AND MISCONCEPTIONS**
$\sqrt{x-1}+3=x-4$ $\sqrt{x-1}=x-4-3$ $\left(\sqrt{x-1}\right)^2=(x-7)^2$ $x-1=x^2-14x-49$ $x^2-15x+50=0$ $(x-5)(x-10)=0$ $x\neq 5\ or\ x=10$	1.Isolation of surds 2.Square both sides 3.standard form 3.Factorization 4.Solve the unknown. 5.Testify the values.	1.Candidates don't isolate the surds. 2.Some candidates don't verify the answer.
$3\sqrt{x}=x-4$ $\left(3\sqrt{x}\right)^2=(x-4)^2$ $9x=x^2-8x+16$ $x^2-17x+16=0$ $(x-16)(x-1)=0$ $x=16\ or\ x\neq 1$		1.candidate always forget to square 3 when they square both sides.

ACTIVITY 4 (LEVEL 2)

4.1 $2-x=\sqrt{2-7x}$ (4)
4.2 $x=1+\sqrt{7-x}$ (5)
4.3 $x-3\sqrt{x+2}=2$ (4)
4.4 $x-3\sqrt{x}-4=0$ (4)
4.5 $\left(\sqrt{x-1}-3\right)\left(\sqrt{x-1}+2\right)=0$ (4)
4.6 $\sqrt{7+3x}+2x=0$ (5)
4.7 $x+3-2\sqrt{5-x}=0$ (5)
4.8 $x-2\sqrt{x-1}=4$ (6)
4.9 $\sqrt{x+5}+1=x$ (3)
4.10 $x-\sqrt{x}=6$ (4)

INEQUALITIES

are mathematical equations involving the symbols such as less than (<) or greater than (>), as well as less or equal to (≤) and greater or equal to (≥).

Example	**Steps to be followed**	**Errors and misconceptions**
$-2x^2+5x+12\geq 0$ $2x^2+5x-12\leq 0$ cv $(2x+3)(x-4)=0$ $x=-\frac{3}{4}\ or\ x=4$ $\therefore -\frac{3}{4}\leq x\leq 4$	Division by negative affect the inequality sign by changing it to the opposite one. Critical values Represent the critical points on a graph or number line then get the values of x	• most candidates forget to change the inequality after dividing by negative. • Most candidates forget to get the critical values first.

ACTIVITY 5

5.1	$(3-x)(x+1)<0$	(4)
5.2	$x^2-64\leq 0$	(4)
5.3	$(x-1)(x-2)\leq 6$	(4)
5.4	$x^2-9x\geq 36$	(4)
5.5	$3x^2+x-2\geq 0$	(4)
5.6	$5x^2+4>21x$	(4)
5.7	$6x-2x^2\leq 0$	(4)
5.8	$4x^2-1<0$	(4)
5.9	$-x^2<-x-12$	(4)
5.10	$-x^2-3x>0$	(4)
5.11	$(x-1)(x+4)\geq 6$	(3)

EXPONENTIAL EQUATIONS

- An equation with a variable in the exponent positions.
- Product rule: $a^m * a^n = a^{m+n}$
- Quotient Rule $\frac{a^m}{a^n}=a^{m-n}$
- Zero Exponent Rule: $a^0=1$
- Negative Exponent Rule $a^{-m}=\frac{1}{a^m}$

Fractional Exponent Rule $a^{\frac{m}{n}}=n\sqrt{a^m}$

Example 1	Explanations	Errors and misconceptions
$\frac{(16)^{\frac{1}{4}}}{(32)^{\frac{1}{5}}}$ $=\frac{(2^4)^{\frac{1}{4}}}{(2^5)^{\frac{1}{5}}}$ $=\frac{2}{2}$ $=1$	Convert 16 and 32 to the lowest prime power. And apply the power raised to another rule.	Learners have a problem of the prime base rule.
Example 2	**Explanation**	Candidates have a problem to apply the power-to-power rule.
$\left(\frac{1}{5}\right)^x=125$ $(5^{-1})^x=5^3$ $-x=3$ $x=-3$	1. Express the bases as the lowest prime numbers. 2. power to power rule. 3.equal bases means equal exponents. 4. divide by negative both sides.	

ACTIVITY 6

6.1 $\sqrt[n]{\left(\frac{10^n+2^{n+4}}{5^{2n}+4(5^n)}\right)}$ simply completely without the use of a calculator (4)

6.2 $3^{x+2} - 3^{2-x} = 82$ (5)

6.3 $3^{x+1} - 3^{x-1} - 24 = 0$ (5)

6.4 $2^{2x} - 6(2^x) = 16$ (4)

TOPIC: ALGEBRA LEVEL 3 AND 4

EXAM GUIDELINE

- Solving quadratic equations by completing the square will NOT be examined.
- Solving quadratic equations using the substitution method (k-method) is examinable.
- Equations involving surds that lead to a quadratic equation are examinable.
- Solution of non-quadratic inequalities should be seen in the context of functions.
- The nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

Simultaneous equations

- Is a finite set of equations for which common solutions are sought.

Examples	Explanation	Error and misconceptions
$3^{x-10} = 3^{3x} --(1)$ $y^2 + x = 20 --- (2)$ $x - 10 = 3x$ $-2x = 10$ $x = -5$ $y^2 + x = 20$ $y^2 - 5 = 20$ $y^2 = 25$ $\sqrt{y^2} = \sqrt{25}$ $y = \pm 5$	Equal bases, equal exponents Solve for x Sub eq 1 value of x into eq 2 Then solve for y	• Most candidates fail to see that they have to use the law of exponents. • Most candidates fail to say y is $\pm$ after using a square root.
$x = 2y ----(1)$ $x^2 - 5xy = -24—(2)$ $sub\ eq1\ into\ eq\ 2$ $(2y)^2 - 5y(2y) = -24$ $4y^2 - 10y^2 = -24$ $-6y^2 = -24$ $y^2 = 4$ $y = \pm 2$	Follow the same steps	• Most candidate they always fail to choose which one has the lowest coefficient between x and y.

QUESTION 1 (LEVEL 3)

1	Solve for x: $\dfrac{\sqrt{10^{1009}}}{\sqrt{10^{1011}} - \sqrt{10^{1007}}} = x$	(3)
2	Simplify: $\left(\dfrac{\sqrt{3^{2011}}-\sqrt{3^{2009}}}{\sqrt{3^{2008}}} + \sqrt{3}\right)^2$	(4)
3	$\sqrt{\dfrac{7^{2014} - 7^{2012}}{12}} = a(7^b)\ calculate\ \ a\ and\ b\ \ , a\ is\ not\ a\ multiple\ of\ 7$	(4)
4	Solve for $\left(\sqrt{\sqrt{2} - x}\right)\left(\sqrt{\sqrt{x} + 2}\right) = x$	(5)
6	$2^x(x - 5) \leq 0$	(3)
7	Given the equation $x^2 + 2xy - 8y^2 = 0$	

	7.1 determine the values of the ratio $\frac{x}{y}$. 7.2 Hence, Determine the values of x and y if x + y =6	(3) (5)
8	Given: f(x) = $3(x-1)^2+5$ and G(x) = 3 8.1 Is it possible for f(x) =g(x)? justify your answer	(2)
9	Given: $x+\frac{1}{x}=4$ 9.1 Determine the value of $x^2+\frac{1}{x^2}$ 9.2 Determine the value of $x^3+\frac{1}{x^3}$	(2) (3)
	Simultaneous equations:	
10	Solve for both x and y simultaneously	
10.1	$(3x-y)^2+(x-5)^2=0$	(6)
10.2	$2x-y=3$ $x^2+5xy+y^2=15$	(6)
10.3	$x+y=2$ $6x+5xy-5y=8$	(3)

QUESTION 2 LEVEL 4

	Solve for x	
1	$6^x+6^x+6^x+6^x+6^x+6^x=6^{6x}$	(3)
2	$x^{\frac{1}{2}}+3x^{\frac{1}{4}}-18=0$	(4)
3	$3^{x+1}+m.3^x=2m+6$	(4)
4	$9.3^{2x}+1=6.3^x$	(4)
5	**Solve for x and y**	
5.1	$2y=3+x$ and $2xy+7=x^2+4y^2$	(3)
5.2	$(3x-y)^2+(x-5)^2=0$	(3)
5.3	$2^{x+1}+2^x=3^{y+2}-3^y$	(3)
5.4	$2^x-2^{y+2}=0$ and $x^2+2xy+y^2$	(3)
6	Given: $f(x)=3(x-1)^2+5$ and $g(x)=3$	
6.1	Is it possible for $\boldsymbol{f(x)}=\boldsymbol{g(x)}$? Justify your answer	(3)
6.2	$Determine\ the\ value(s)\ of\ k\ for\ which\ f(x)=g(x)+k$ has TWO equal roots.	(3)
7	Consider the product: $1\times2\times3\times4\times\ .\ .\ .\ \times30$. Determine the largest value of (k) such that 2^k is a factor of this product.	(3)
8	The equations $x^2+ax+b=0$ and $x^2+bx+a=0$ both have real and equal roots. Solve for a and b, where a >0 and b > 0.	(7)

PATTERNS, SEQUENCES

AND

SERIES

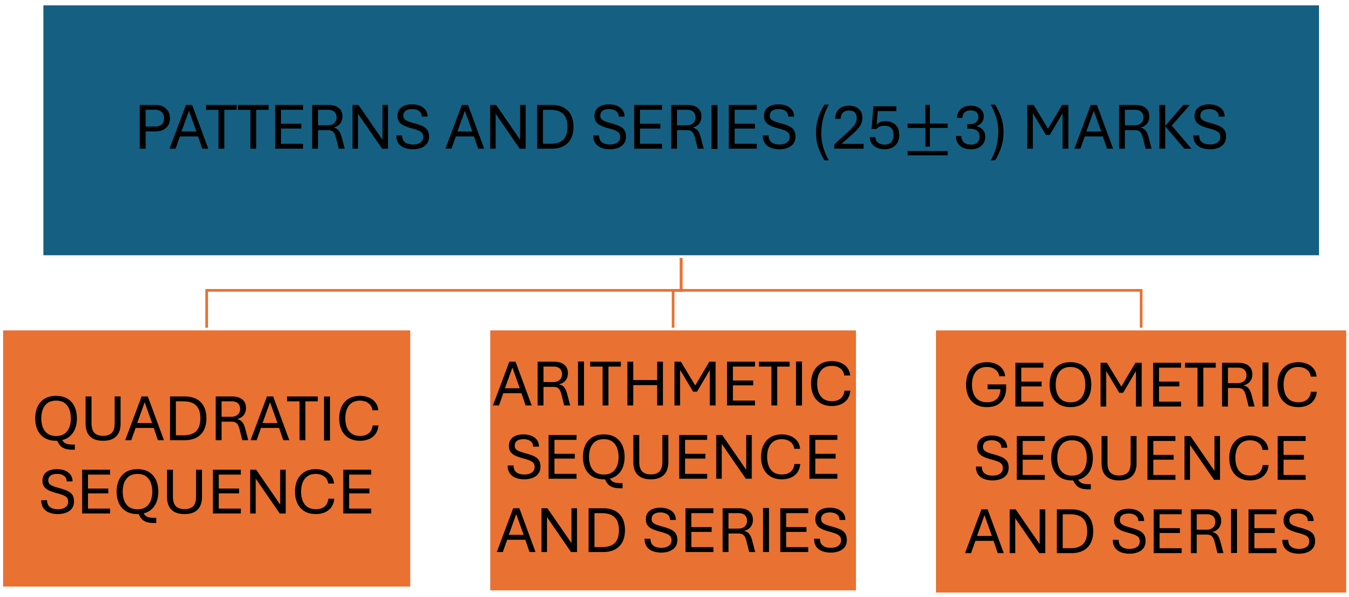
ACTIVITY MANUAL

COGNITIVE LEVEL 1 & 2 QUESTIONS

WHAT IS A NUMBER PATTERN?

A NUMBER PATTERN IS AN ORDERED LIST OF NUMBERS OR OBJECTS.

COMPOSITION OF PATTERNS AND SERIES



CONCEPT	**NOTES**	**FORMULA**
Quadratic sequence	• Is a sequence of numbers in which the second difference between any two consecutive terms is constant • The first difference is calculated by taking the difference between consecutive terms e.g $T_1 = a + b + c$ $T_2 = 4a + 2b + c$ $\therefore T_2 - T_1 = 3a + b$ • Second differences is obtained by taking the difference between consecutive first differences • $2a = d_2$	$T_n = an^2 + bn + c$
Arithmetic sequence	• Is a sequence in which the first difference between consecutive terms is constant • The general term formula allows you to determine any specific term of an arithmetic sequence • n is the position of the term	$T_n = a + (n-1)d$ $d = T_n - T_{n-1}$ or $d = T_{n+1} - T_n$

	• T_n is the value of a specific term/the last term	
Arithmetic series	• Is the sum of the terms of an arithmetic sequence e.g $T_1 + T_2 + T_3$	$S_n = \frac{n}{2}[2a - (n-1)d]$ or If given the last , then $S_n = \frac{n}{2}(a + l)$
Geometric sequence	Is a sequence of numbers in which there is a constant ratio between consecutive terms	$T_n = ar^{n-1}$ $r = \frac{T_n}{T_n - 1}$ or $\frac{T_{n+1}}{T_n}$
Geometric series	It is the sum of n terms of a geometric sequence, where n is the number of terms	$S_n = \frac{a(r^n - 1)}{r - 1}, r > 1$ OR $S_n = \frac{a(1 - r^n)}{1 - r}, r < 1$
Convergent geometric series	• It is a series in which the terms get smaller and smaller as we add more terms. • The series is said to be convergent if the constant ratio is between -1 and 1	$-1 < r < 1$
Sum to infinity	• It is the result of adding all of the terms in an infinite geometric series together • It is only possible to calculate the sum to infinity for geometric series that converge	$S_\infty = \frac{a}{1 - r}$

Sigma notation	• It is a useful shorthand method to indicate the sum of terms in a series • The symbol $\sum$ indicates the sum	$\sum_k^n T_n$ n is the last term k is the first term $n = top - bottom + 1$

EXAMINATION GUIDELINE

1. The sequence of the first difference of a quadratic number pattern is linear. Therefore, knowledge of a linear pattern can be constant in the context of a quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

COMMON ERRORS AND MISCONCEPTIONS

COMMON ERROR	SUGGESTION FOR IMPROVEMENT
Learners use incorrect formulae	Learners must be made aware of which formulae on the information sheet apply to which type of sequence. It is good practice for them to use the information sheet in class so that they become familiar with it.
Learners are not able to solve questions which require the use of simultaneous equation.	Educators should emphasise the teaching of simultaneous equations as it is taught in algebra.
Learners are not able to differentiate between the n^{th} term, sum of n terms, sum to infinity formulae in arithmetic or geometric sequences and series	Educators should emphasise the differences between the n^{th} terms, sum and sum to infinity formulae in arithmetic and geometric sequence.
Learners are unable to determine between which two consecutive terms, a term in the first difference lies.	Educators should give more questions on determining the specific values of the term which makes the first difference.
Inability to determine variables' value(s) given in quadratic patterns.	Educators should emphasise more on how to find the numerical values of a, b and c.
Learners view quadratic and arithmetic as two different patterns which do not have a relationship.	Educators should give questions about arithmetic while given quadratic pattern
Failure to understand the meaning of a converging series and its conditions.	Educators should emphasise more on convergent series questions involving determination of a value for a specific variable
Failure to derive a pattern from a word pattern and diagrams	Educators should provide activities involving word problems

COGNITIVE LEVELS 1 AND 2 QUESTIONS

QUESTION 1

Consider the finite linear number pattern: 20; 17; 14; …; −103.

1.1 Write down the 4th term of the pattern. (1)

1.2 Determine the expression for the n^{th} pattern. (2)

1.3 Calculate the number of terms in the sequence. (2)

QUESTION 2

Given the quadratic sequence: 3; 5; 11; 21; …

2.1 Write down the value of the next term, if the pattern continues. (1)

2.2 Determine the value of the 48th term. (5)

QUESTION 3

Given the arithmetic sequence: 3; b ;19; 27…

3.1 Calculate the value of b. (2)

3.2 Determine the n^{th} term of the sequence. (3)

3.3 Calculate the value of the thirtieth term. (2)

3.4 Calculate the sum of the first 30 terms of the sequence. (3)

QUESTION 4

4.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$. (4)

4.2 The following arithmetic sequence is given: $\mathbf{20}; \mathbf{23}; \mathbf{26}; \mathbf{29}; \dots ; \mathbf{101}$

4.2.1 How many terms are there in this sequence? (2)

QUESTION 5

The sequence 4; 9; x; 37; … is a quadratic sequence.

5.1 Calculate x. (3)

5.2 Hence, or otherwise, determine the n^{th} term of the sequence. (4)

QUESTION 6

The sequence 3; 9; 17; 27; … is a quadratic sequence.

6.1 Write down the next term. (1)

6.2 Determine an expression for the n^{th} term of the sequence. (4)

QUESTION 7

7.1 Given the sequence 4; x; 32. Determine the value(s) of the x if the sequence is:

7.1.1 Arithmetic. (2)

7.1.2 Geometric. (3)

7.2 Determine the value of P if: $P = \sum_{k=1}^{13} 3^{k-1}$. (4)

7.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

QUESTION 8

Given the arithmetic series: $-7 - 3 + 1 + \cdots + 173$

8.1 How many terms are there in the series? (3)

8.2 Calculate the sum of the series. (3)

8.3 Write the series in sigma notation. (3)

QUESTION 9

9.1 Consider the geometric sequence: 4; −2; 1…

9.1.1 Determine the next term of the sequence. (2)

9.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

9.1.3 Calculate the sum to infinity of the series $4 - 2 + 1\ldots$ (2)

9.2 If x is a real number, show that the following sequence can NOT be geometric:

$1; x + 1; x - 3\ldots$ (4)

QUESTION 10

10.1 Consider an arithmetic series: $S_n = n^2 + 3n$

10.1.1 Calculate the second term of this series. (3)

10.1.2 The first two terms of this sequence form a geometric sequence.

Calculate the sum of the first 6 terms of this geometric sequence. (4)

10.2 Calculate: $\sum_{n=1}^{\infty} 6(\frac{1}{2})^n$ (4)

QUESTION 11

The general term of the sequence is given by $T_n = an^2 + bn + c$

$T_2 - T_1 = 4; T_3 - T_2 = 6;\ T_4 - T_3 = 8\ and\ T_2 = 6$

11.1 Calculate a, b and c (4)

11.2 Determine the 12^{th} term of the sequence. (2)

QUESTION 12

A geometric series has a constant ratio of $\frac{1}{2}$ and the sum to infinity of 6. Calculate:

12.1 The first term of the series. (2)

12.2 The 8^{th} term of the series. (2)

12.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5.8125$ calculate the value of *n*. (4)

QUESTION 13

Given the geometric sequence: $-\frac{1}{4}$, b, -1;…

13.1 Calculate the possible values of *b*. (3)

13.2 If $b = \frac{1}{2}$ calculate the 19th term (T_{19}.) of the sequence. (3)

13.3 If $b = \frac{1}{2}$ write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

13.4 Is the geometric series formed in QUESTION 13.3 convergent?
Give reason for your answer. (2)

QUESTION 14

6; 6; 9; 15; … are the first four terms of a quadratic number pattern.

14.1 Write down the value of the fifth term (T_5) of the pattern. (1)

14.2 Determine the formula to represent the general term of the pattern. (4)

14.3 Which term of the pattern has a value of 3 249? (4)

QUESTION 15

The first four terms of a sequence are given:

$\frac{2}{1\times2}$; $\frac{2}{2\times3}$; $\frac{2}{3\times4}$; $\frac{2}{4\times5}$

15.1 Write down the next two terms of the sequence. (2)

15.2 Determine the n^{th} term of the sequence. (2)

15.3 Use the answer to QUESTION 13.2 to calculate n if the n^{th} term is $\frac{1}{66}$ (4)

COGNITIVE LEVELS 3 AND 4

QUESTION 1

1.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$.

Prove without the use of a calculator that the sum of the series to infinity is $16 + 8\sqrt{2}$ (4)

1.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

1.2.1 Write the series in sigma notation. (2)

1.2.2 Calculate the value of x. (3)

QUESTION 2

2.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n + 9)$.

Calculate the 23rd term of the sequence. (5)

2.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)

QUESTION 3

The following sequence is a combination of an arithmetic and geometric sequence:

3; 3; 9; 6; 15; 12;…

3.1 Calculate $T_{52} - T_{51}$. (5)

3.2 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

QUESTION 4

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n, and has the form $T_n = an^2 + bn + c$. The distances are recorded below:

Time (in seconds)	1	2	3	4	5	6
Distance (in meters)	17	10	5	2	r	s

4.1 Determine the values of r and s. (3)

4.2 Determine the values of a, b and c. (4)

4.3 How far is the athlete from P when $n = 8$ (2)

4.4 Show that the athlete is moving towards P when $n < 5$, and away from P when $n > 5$. (4)

QUESTION 5

5.1 6; 6; 9; 15; … are the first four terms of a quadratic number pattern.

5.1.1 Write down the value of the 5^{th} term of the pattern. (1)

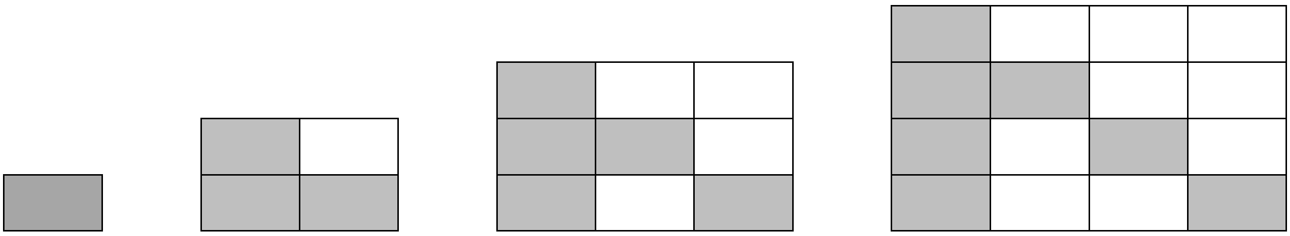
5.1.2 Determine the formula to represent the general term of the pattern. (4)

5.1.3 Which term of the pattern has the value of 3 249? (4)

5.2 Determine the value(s) of x in the interval $x \in [0°; 90°]$ for which the sequence $-1; 2sin3x ; 5;\ldots$ will be arithmetic. (4)

QUESTION 6

Dark tiles (D) and light tiles (L) are used to create patterns on the floor. The first four patterns are shown below. For the patterns that follow, the tiles are arranged in a similar manner.



Pattern 1 **Pattern 2** **Pattern 3** **Pattern 4**

6.1 How many dark tiles were used in pattern 5? (1)

6.2 How many light tiles were used in pattern 6? (1)

6.3 Write down the general term (D_n) for the number of dark floor tiles used in each pattern. (2)

6.4 Write down the general term (D_n) for the number of light floor tiles used in each pattern. (2)

6.5 Which pattern will have exactly 64 light floor tiles? (3)

6.6 Each dark tile is 0,3m wide and 0,6m long. Calculate the area covered by all the dark tiles in the first 100 patterns. (3)

QUESTION 7

The n^{th} term of a sequence is given by: $T_n = -2(n-5)^2 + 18$.

7.1 Write down the first THREE terms of the sequence. (3)

7.2 Which term of the sequence will have the greatest value? (1)

7.3 What is the second difference of this quadratic sequence? (2)

7.4 Determine ALL values of n for which the terms of the sequence will be less than -110.(6)

QUESTION 8

The following number pattern is given: 13; 27; 45; 67;…

8.1 Is this a quadratic pattern? Justify your answer with relevant calculations. (2)

8.2 Determine the general term Tn of the quadratic number pattern. (4)

8.3 Calculate the value of T_{100}. (2)

8.4 The first difference between two consecutive terms of the quadratic number pattern is 110. Determine the values of these two terms. (5)

8.5 Show that ALL the terms of this quadratic number pattern will be odd numbers. (2)

QUESTION 9

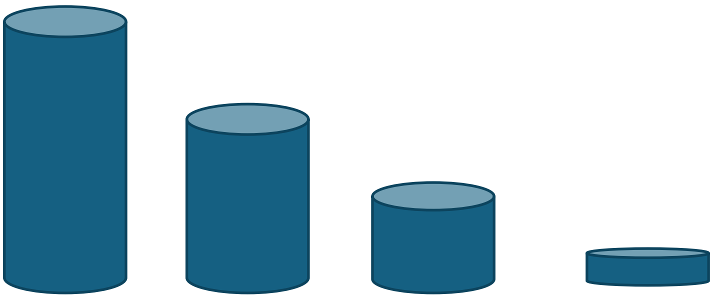
A sequence of squares, each having side 1, is drawn as shown below. The first square is shaded, and the length of the side of each shaded square is half the length of the side of the shaded square in the previous diagram.

9.1 Determine the area of the unshaded region in diagram 3. (2)

9.2 What is the sum of the areas of the unshaded regions of the first 7 squares? (5)

QUESTION 10

Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water. Would it be possible for first water tank to hold all the water from the other 19 tanks? Motivate your answer through calculations.



(5)

QUESTION 11

Consider the number pattern below created by using the numbers of the sequence:

$$2 \;;\; 6 \;;\; 10 \;\; 14 \;;\; 18 \;;\; \ldots$$

2

6 10

14 18 22

26 30 34 38

42 … … … …

11.1 Calculate the sum of the numbers in the 8th row. (3)

11.2 Determine the mean of the numbers in the 20th row. (2)

QUESTION 12

Consider the sequence: $\frac{1}{2}\;;\;4\;;\;\frac{1}{4}\;;\;7\;;\;\frac{1}{8}\;;\;10\;;\;\ldots$

12.1 Write down the next TWO terms of the sequence. (2)

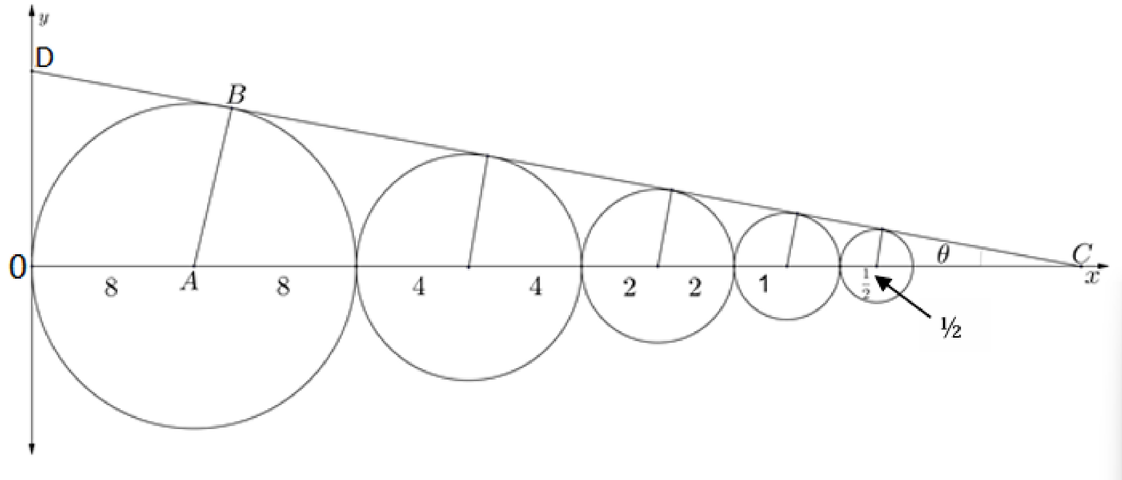
12.2 Calculate the sum of the first 51 terms of the sequence. (4)

QUESTION 13

Each time a photocopy is made from a previous photocopy, the quality of the print decreases by 11%. Determine how many times this photocopying can be done before the quality becomes less than 20% of the original. (4)

QUESTION 14

An infinite number of circles, touching each other is drawn below. The diameters of the circles are on the line OC. The length of the radius of the circle A is 8 units. The radius of each circle thereafter is half the length of the radius of the previous circle. Straight line DC is a tangent to all the circles.



14.1 Calculate the length of the radius of the 15th circle. (3)

14.2 Show that OC = 32 units. (3)

14.3 Write down the size of angle ABC. (1)

14.4 Calculate the value of $\tan\theta$ (leave your answer in the simplest surd form) (3)

THE END

TOPIC: FUNCTIONS
SUB-TOPICS

- STRAIGHT LINE/LINEAR
- PARABOLA/QUADRATIC
- HYPERBOLA
- EXPONENTIAL
- INVERSE AND LOGARITHIMIC FUNCTIONS

EXAMINATION GUIDELINE

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x), -f(x), f(x+a), f(x)+a, af(x) and\ x = f(y) where\ a \in R$.
2. Trigonometric functions will ONLY be examined in PAPER 2.

DIAGNOSTIC REPORT
MISCONCEPTIONS AND ERRORS

(a) Learners do not calculate the coordinates of the x and y intercepts correctly which results in an incorrect equation.
(b) Inability to determine the maximum vertical distance between two functions, especially when the straight line is above the parabola.
(c) Learners fail to determine the equation of inverse of a function, and how to sketch it.
(d) Failure to determine domain and range of an inverse function from the exponential function without sketching.
(e) Failure to determine the value(s) of the variable(s) in the equation when given a sketch of a function.
(f) Failure to determine roots of a function, in particular positive and negative roots.
(g) Learners are failing to interpret notation involving inequalities.
(h) Learners believe that the log function is the inverse of all functions.

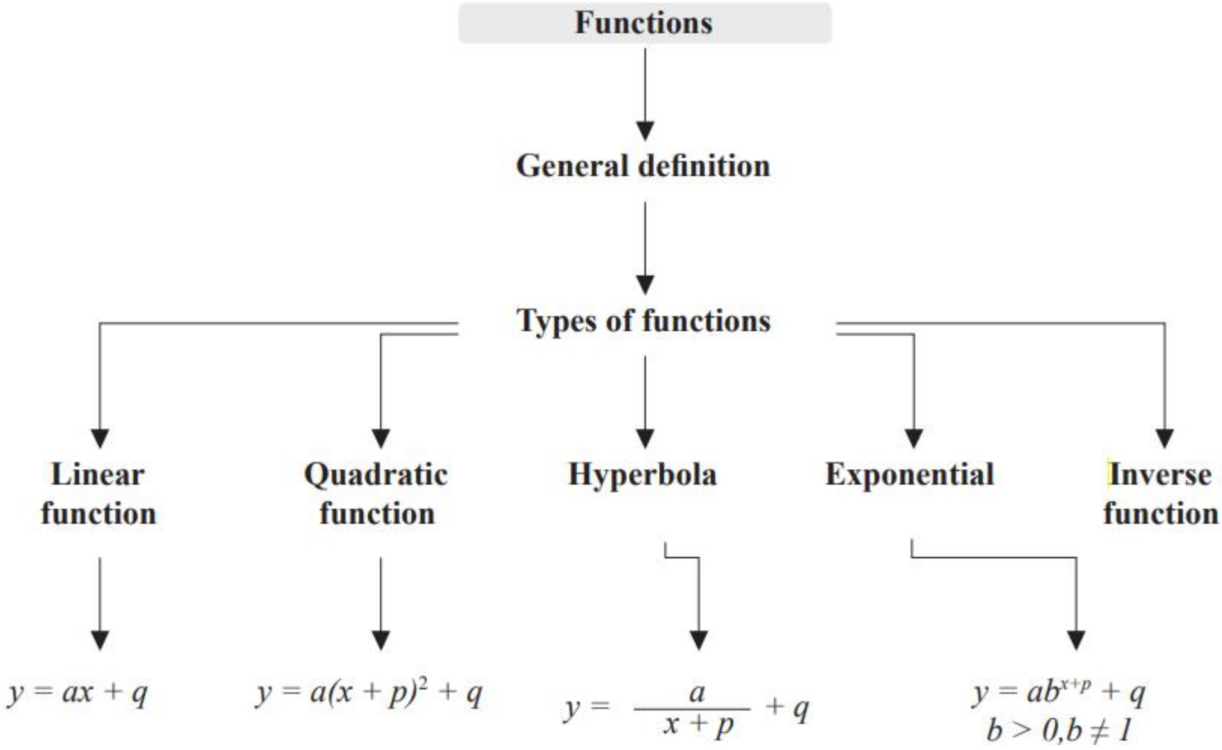
SUGGESTIONS FOR IMPROVEMENT

(a) Teachers need to emphasize what the points on a graph are, and help learners identify what properties of these points are, before they start answering a question.
(b) Vertical and horizontal distances should be explained without the use of the distance formula when interpreting functions. In conjunction with this, teachers must emphasize that distance is a scalar property and cannot be negative.
(c) Teachers should work with restricted domain graphs in class to make learners aware that functions can be restricted and emphasize the effect this has on their properties.
(d) Teachers should spend some time on graphical interpretation of functions. This can be started with the very first graph that is sketched in Grade 10.

(e) The concepts of $f(x) > 0$, $f(x).g(x) > 0$ must be emphasized throughout the FET phase when teaching functions.
(f) The link for learners between the algebraic work (i.e. nature of roots, simultaneous equations and inequalities) and the graphical representation must be created by the teacher when

working with functions. However, teachers need to emphasize the importance of understanding these concepts and teach the learners to read off the solutions to the questions from the graph. Not all solutions in functions questions need be algebraic – this practice seems to be the default for most candidates.

(g) Give more questions in which the straight line, parabola, exponential and hyperbola are drawn and learners are required to determine its equation.



DEFNITIONS:

• Asymptotes: These are the lines that a graph ‘tends towards’ but doesn’t reach.

• A function is increasing if the values of y increase as x values increase.

• A function is decreasing if the values of y decrease as x values increase or vice versa.

• Domain: These are x-values for which the graph is defined or a set of values assigned to the independent variables of a function

• Range: A set of values that a function can take for all possible y-values.

• A function is a relationship between x and y, where for every x-value there is only one y-value.

•Axis of symmetry is an imaginary straight line that divides a shape into two identical parts, thereby creating one part as the mirror imagine of the other part.

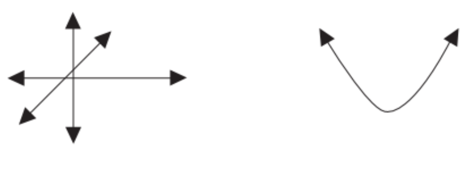
Properties of functions:
Axis of symmetry
Domain
Range
Notation

$y = ax + q$

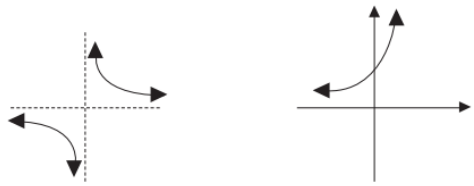
$y = a(x + p)^2 + q$

$y = \dfrac{a}{x + p} + q$

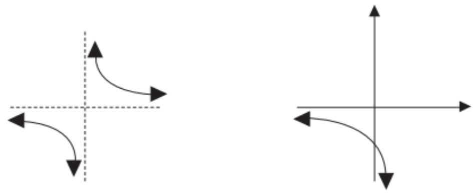
$y = ab^{x+p} + q$
$b > 0, b \neq 1$



a* > *0



a* > *0



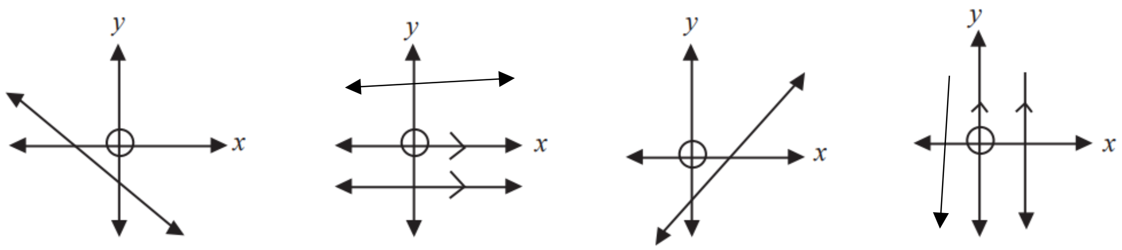
STRAIGHT LINE or LINEAR FUNCTION

General representation or equation

y = ax + q or y = mx + c

a or m is the gradient and q or c is the y - intercept

Also note the shape of the following linear functions:



$a < 0$
$q < 0$

$a = 0$
$y = q$

$a > 0$
$q < 0$

a is undefined
there is no *q*-value

Domain and range is $x \in \mathbb{R}$ and $y \in \mathbb{R}$ respectively.

PARABOLA or QUADRATIC FUNCTION

Note:

- Solving quadratic equations, is a pre-knowledge, in order to be able to work with a parabola.
- The standard form of a quadratic funcion is $\boldsymbol{y = ax^2 + bx + c}$
- **I**f a quadratic equation is already factorized and one side is equal to zero, then the equation is in standard form and you should just write the answers from the given factors.
- If one side is factorized and the other side is not equal to zero, then frst write the equation in standard form and then factorize.
- When the question says correct to one or two decimal places, you are expected to solve the quadratic equation using the quadratic formula.
- If you struggle to fnd factors of a quadratic equation by inspection, use the quadratic formula
- For the inequality, simplify so that the right-hand side is 0.
- Then use the graphic (draw the sketch of the parabola) or number line method.

GENERAL REPRESENTATION OR EQUATION

$y= a(x+p)^2 +q$ or $y = ax^2 + bx + c$ or $y = a(x - x_1)(x - x_2)$

Important deductions

for $a < 0$

for $a > 0$

- For $y = ax^2$, $p = 0$ and $q = 0$, the **turning point** is (0;0) and **y-intercept** is $y = 0$

The domain is $x \in \mathbf{R}$ and **the range** is $y \geq 0; y \in \mathbf{R}$ if $a > 0$ or $y \leq 0; y \in \mathbf{R}$ or R if $a < 0$

- For $y = ax^2 + q$, $p = 0$, the turning point is (0;q) and y-intercept is $y = q$

The domain is $x \in \mathbf{R}$ and **the range** is $y \geq q; y \in \mathbf{R}$ if $a > 0$ or $y \leq 0; y \in \mathbf{R}$ if $a < 0$

- For $y = a(x + p)^2 + q$, the **turning point** is (-p; q) and **y-intercept** is $y = a(p)^2 + q$ The domain is $x \in \mathbf{R}$ and **the range** is $y \geq q; y \in \mathbf{R}$ if $a > 0$ or $y \leq q; y \in \mathbf{R}$ if $a < 0$
- For $y = ax^2 +bx +c$, the turning point is $\left(\frac{-b}{2a}; \frac{4ac - b^2}{4a}\right)$ and y-intercept is $y = c$
- The **domain** is $x \in \mathbf{R}$ and the **range** is $y \geq \frac{4ac - b^2}{4a}$; $y \in R$ if $a > 0$ or $y \leq \frac{4ac - b^2}{4a}$; $y \in R$ if $a < 0$
- The roots or x-intercepts are determined by equating y=0 and solving for x.

SKETCHING THE GRAPH OF A PARABOLA, YOU NEED:

• the y-intercept (let x = 0)

• the x-intercepts (let y = 0)

• the axis of symmetry is ($x = \frac{-b}{2a}$) if the equation is of the form $f(x) = ax^2 + bx + c$

or (x = p) if the equation is of the form $f(x) = a(x - p)^2 + q$ or

$x = \frac{x_1+x_2}{2}$ if the equation is of the form $f(x) = a(x - x_1)(x - x_2)$

• the maximum/ minimum value is obtained by substituting the axis of symmetry in the given function.

- Turning point is obtained as $[(\frac{-b}{2a};\ f\left(\frac{-b}{2a}\right)$ or $\frac{-(b^2-4ac)}{4a}]$ if the equation is in the form of $f(x) = ax^2 + bx + c$
- Turning point can also be obtained as $(p; q)$ if the equation is of the form $f(x) = a(x - p)^2 + q$
- Turning point can also be obtained as $[(\frac{x_1+x_2}{2}; f(\frac{x_1+x_2}{2})]$ if the equation is of the form $f(x) = a(x - x_1)(x - x_2)$

FINDING THE EQUATION OF A GIVEN PARABOLA:

• given the roots x_1 and x_2 and one other point use $y = a(x - x_1)(x - x_2)$

• given the turning point (p;q) and one other point, use $y = a(x - p)^2 + q$

• given three points on the on the graph, with one being the y- intercept, substitute the other two points into $y = ax^2 + bx + c$ and solve the equations simultaneously.

EXAMPLE

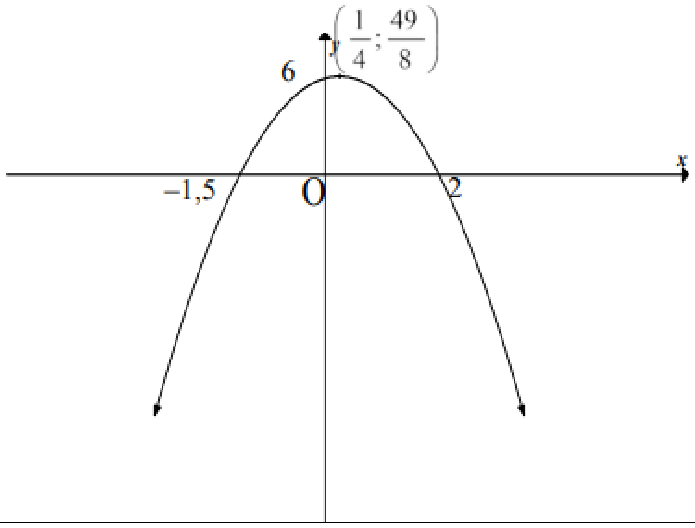
QUESTION 5

Given: $f(x) = -2x^2 + x + 6$

5.1	Calculate the coordinates of the turning point of f.	(4)
5.2	Determine the y-intercept of f.	(1)
5.3	Determine the x-intercepts of f.	(4)
5.4	Sketch the graph of f showing clearly all intercepts with the axes and turning point.	(3)
5.5	Determine the values of k such that $f(x) = k$ has equal roots.	(2)
5.6	If the graph of f is shifted two units to the right and one unit upwards to form h, determine the equation h in the form $y = a(x+p)^2 + q$.	(3)
		[17]

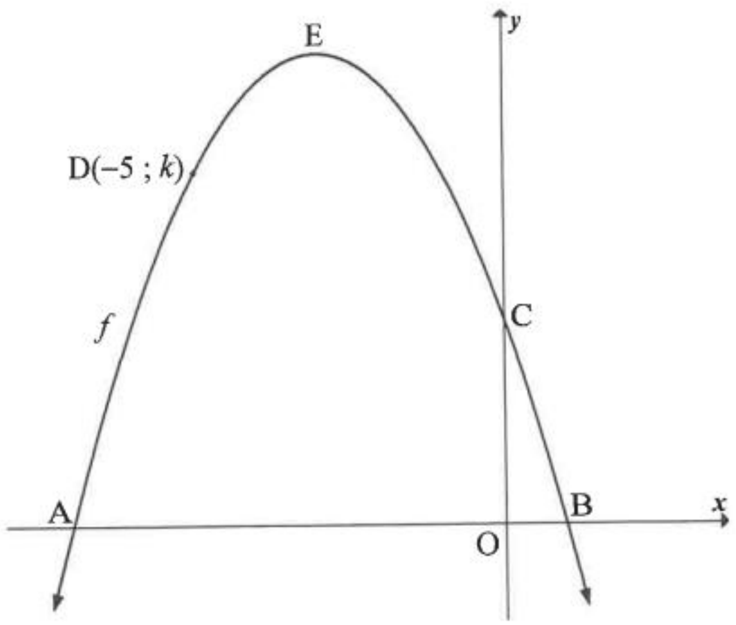
QUESTION 5

5.1	$x = -\frac{b}{2a}$ $= -\frac{1}{2(-2)}$ $= \frac{1}{4}$ $\therefore y = -2\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right) + 6$ $y = \frac{49}{8}$	✓substitution/*verv.* ✓x-value/*wrde* ✓substitution/*verv.* ✓ y-value/*wrde* (4)
5.2	$y = -2(0)^2 + 0 + 6$ $\therefore y$ intercept $(0;6)$	✓ y-value/*wrde* (1)
5.3	x intercepts $0 = -2x^2 + x + 6$ $0 = 2x^2 - x - 6$ $0 = (2x+3)(x-2)$ $\therefore x = 2$ or $x = -\frac{3}{2}$ $(2;0)$ and $\left(-\frac{3}{2};0\right)$	✓ $y = 0$ ✓ factorisation/*fakt.* ✓ ✓ x-values/*wrde* (4)

5.4		✓ shape/*vorm* ✓ intercepts/*afsnitte* ✓ turning point/*drpnt* (3)
5.5	$k=\frac{49}{8}$	✓✓ answer/*antw.* (2)
5.6	New/*Nuwe* turning point/*drpn.t* $\left(\frac{9}{4};\frac{57}{8}\right)$ Equation/*verg.* of h $y=-2\left(x-\frac{9}{4}\right)^2+\frac{57}{8}$	✓ ✓ turning points/*drpnt* ✓ equation/*verg.* (3) **OR**/*OF* ✓✓✓ answer only (3) **[17]**

QUESTION 6

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5\,;\,k)$ is a point on f.



6.1	Calculate the coordinates of E, the turning point of f.	(3)
6.2	Write down the value of k.	(1)
6.3	Determine the equation of the straight line passing through C and D.	(4)
6.4	A tangent, parallel to CD, touches f at P. Determine the coordinates of P.	(4)
6.5	For which values of x will $f(x) - 12 > 0$?	(2) **[14]**

QUESTION 6

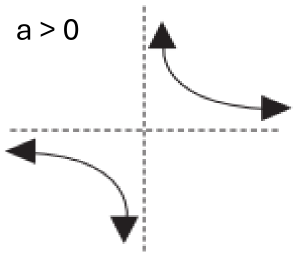
6.1	$f(x)=-x^2-6x+7$ $f'(x)=-2x-6$ $-2x-6=0$ **OR/*OF*** $x=-\frac{(-6)}{2(-1)}$ $x=-3$ E(–3 ; 16) ANSWER ONLY: FULL MARKS	✓ method ✓ x-value ✓ y-value (3)
6.2	$k=f(-5)$ $k=-(-5)^2-6(-5)+7$ $\therefore k=12$	✓ answer (A) (1)
6.3	C(0 ; 7) D(–5 ; 12) $m_{CD}=\frac{12-7}{-5-0}$ $m_{CD}=-1$ Equation of CD: $y=-x+7$	✓ coordinates of C ✓ substitution ✓ m ✓ answer (4)
6.4	$-2x-6=-1$ $-2x=5$ $x=-\frac{5}{2}$ $y=f\left(\frac{-5}{2}\right)=-\left(\frac{-5}{2}\right)^2-6\left(\frac{-5}{2}\right)+7=\frac{63}{4}=15,75$ $\therefore P\left(-\frac{5}{2};\frac{63}{4}\right)$	✓ $f'(x)=-2x-6$ ✓ equating to -1 ✓ x-value ✓ y-value (A) (4)
6.5	Point by symmetry: (–1 ; 12) $-5<x<-1$ **OR/*OF*** $-x^2-6x+7>12$ $-x^2-6x-5>0$ $x^2+6x+5<0$ $(x+1)(x+5)<0$ $-5<x<-1$ ANSWER ONLY: FULL MARKS	✓ -1 ✓ answer (2) ✓ -1 ✓ answer (2)
		[14]

HYPERBOLA

General representation or equation

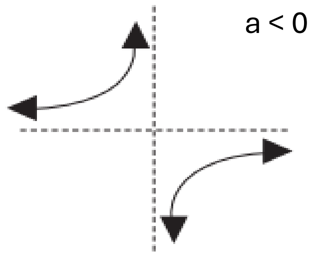
$y = \frac{a}{x-p} + q \quad , a \neq 0$

a > 0



Dotted lines are asymptotes

a < 0



Dotted lines are asymptotes

- q is the vertical translation
- p is the horizontal translation
- General equation of hyperbola: $y = \frac{a}{x-p} + q$,$a \neq 0$
- the vertical asymptote is x = p and the horizontal asymptote is y = q.
- The axis of symmetry is y = ± (x + p) + q

Domain is $x \in \boldsymbol{R}, x \neq p$

Range is $y \in \boldsymbol{R}, y \neq q$

SKETCHING THE GRAPH OF A HYPERBOLA, YOU NEED:

- the y-intercept (let $x = 0$)
- the x-intercepts (let $y = 0$)
- the vertical asymptote is $x - p = 0$ and the horizontal asymptote is $y = q$.

EXAMPLE 3

Given $f(x) = \frac{3}{x-2} + 1$

3.1 Write down the equations of the asymptotes of f.

3.2 Determine: the coordinates of B; the x-intercept of f.

3.3 Determine: the coordinates of D; the y-intercept of f.

3.4 Determine the domain and the range of f.

3.5 Determine the decreasing and increasing functions of the axes of symmetry of f.

3.6 Draw a sketch graph of f.

Solution

3. 1 $y = 1$

$x = 2$

3.2 $y = \frac{3}{x-2} + 1$

$0 = \frac{3}{x-2} + 1$

$-x + 2 = 3$

$x = -1$

3.3 $y = \frac{3}{0-2} + 1$

$y = -\frac{1}{2}$

3.4 Domain: $x \in R, x \neq 2$

Range: $y \in R\ y \neq 1$

3.5 Increasing axis of symmetry

$y = x + c$
$1 = 2 + c$
$c = -1$
$y = x - 1$

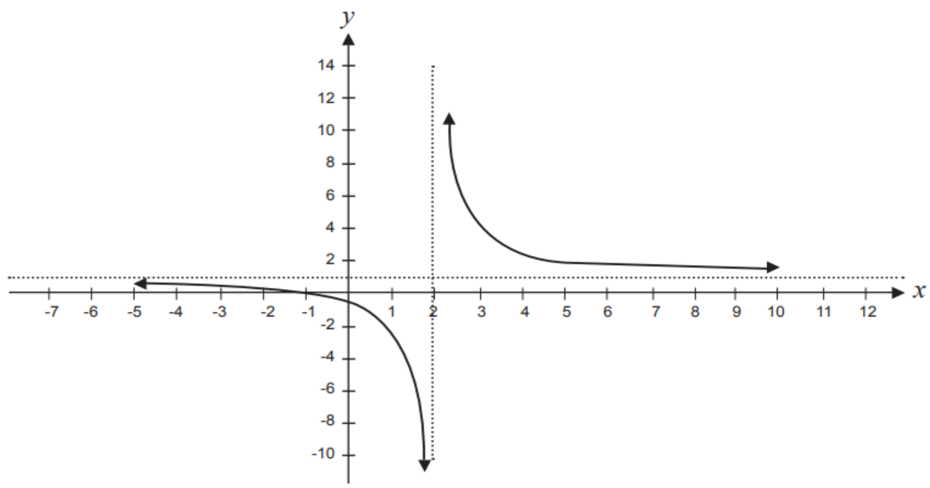
Decreasing axis of symmetry:

y = x + c

1 = -2 + c

y = - x + 3

3.6



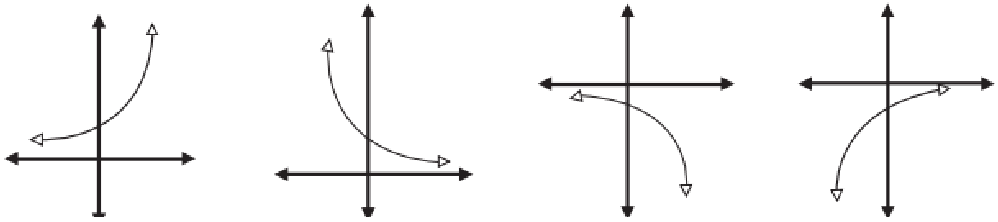
EXPONENTIAL

General representation or equation:

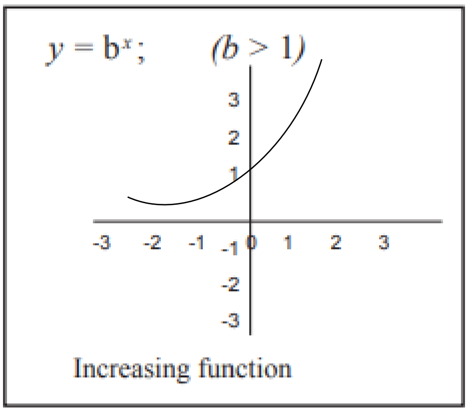
$y = ab^{x+p} + q$, where $a \neq 0, b > 0;\ b \neq 1$

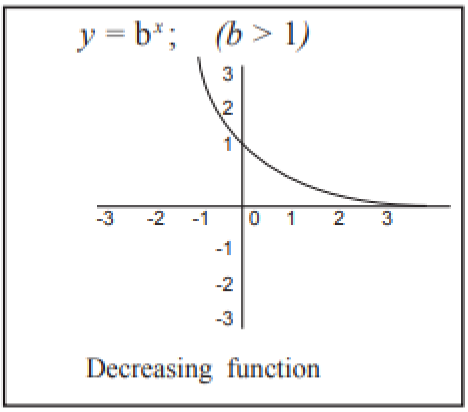
IMPORTANT DEDUCTIONS

for $a < 0$ and $0 < b < 1$ for $a > 0$ and $b > 1$ for $a > 0$ and $0 < b$ for $a < 0$ and $b > 1$



- For $y = ab^{x+p} + q$, the asymptote is $y = q$ and y-intercept is $y = a + q$





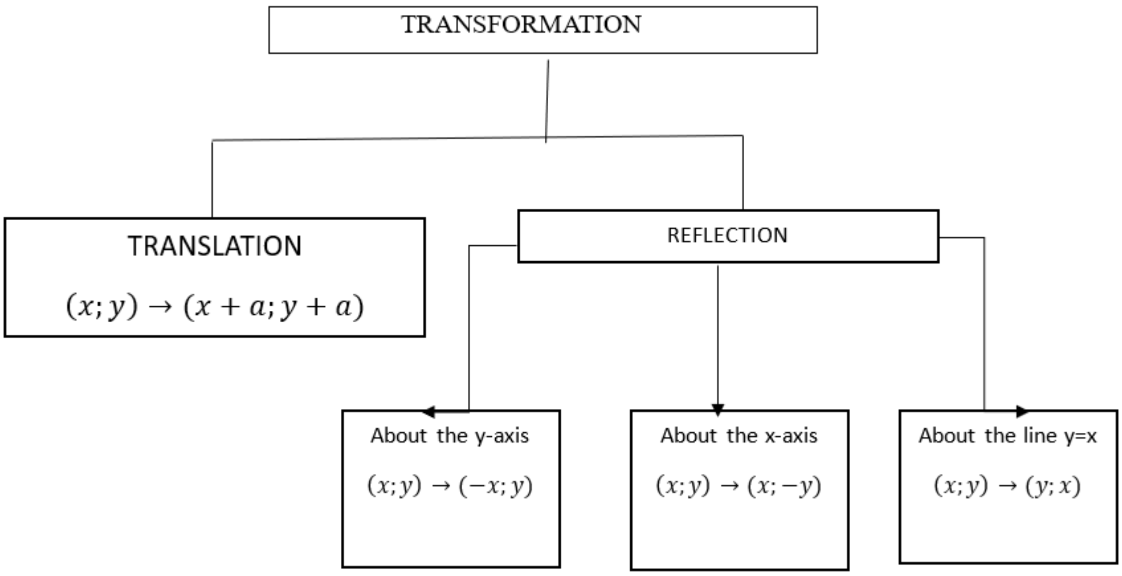
PROCEDURE FOR DRAWING GRAPHS:

- Write down the asymptotes.
- Draw the asymptotes on the set of axes as dotted lines.
- Determine the x- intercept(s); let $y = 0$
- Determine the y- intercept(s); let $x = 0$
- Plot the points; then draw the graph using free-hand.

INTERPRETATION OF GRAPHS

$\boldsymbol{f'(x) > 0}$ {Where the graph is increasing/positive gradient}
$\boldsymbol{f'(x) < 0}$ {Where the graph is decreasing /Negative gradient}
$\boldsymbol{f(x).g(x) > 0}$ {Where both graphs are above or below the x-axis}
$\boldsymbol{f(x).g(x)} < 0$ {one graph above and the other below the x-axis}
$\boldsymbol{f(x) > 0}$ {above the x-axis }
$\boldsymbol{f(x) < 0}$ {below the x-axis }
$\boldsymbol{g'(x) = 0}$ {x-value of the turning point}
$\boldsymbol{g'(x) > 0 \text{ and } f'(x) > 0}$ {both graphs are increasing}

$\boldsymbol{g'(x) < 0}$ ***and*** $\boldsymbol{f'(x) < 0}$ {both graphs are decreasing}
$\boldsymbol{f(x) - g(x) = 1}$ {Graph f(x) is above graph g(x) and the distance is 1 unit}
$\boldsymbol{xf(x)} > 0$ {$+x$ and $+y$ -value or $-x$ and $-y$ -value}
$\boldsymbol{f(x).f'(x)} < 0$ {+y value and a decreasing function or $-y$ -value and increasing function}



INVERSE FUNCTIONS

- A function is a relationship between x and y, where for every x-value there is only one y-value.
- One way to decide whether a graph represents a function is to use the vertical line test.
  If any line drawn parallel to the y-axis cuts the graph only once, then the graph represents a function.
- The term inverse means: reflection about the line $y = x$
- Swap x and y and hence make y be the subject of the formula
- The notation for the inverse of a function is f^{-1}
- The inverse function of a linear is also a linear function
- The inverse of a parabola is not a function, unless the domain of the original function is restricted
- Restriction of parabolic inverse to be a function:
- For Domain: $x \geq 0$ and for the Range: $y \geq 0$
- For Domain: $x \leq 0$ and for the Range: $y \leq 0$

- For parabola : $y = ax^2 \ to \ y = \pm\sqrt{\frac{x}{a}}$
- However, if the domain of the original function is restricted then the inverse will be a function too.
- For example f(x) = x^2, then the inverse of f, $\mathbf{f^{-1}(x) = \pm\sqrt{x}}$, is not a function. But if f(x) has a restricted domain **x ≥ 0 or x ≤ 0**, then the inverse will be a function.
- The inverse of an exponential function is a logarithmic function
- For logarithm: $y = a^x \ to \ y = \log_a x$
- Only use the log for exponential graph or if the variable is an exponent

EXAMPLE

Given $f(x) = 2^x$

(a) Write down the domain and range of f.

(b) Sketch the graph of f showing intercept(s) with axes.

(c) Write down the equation of the graph of g, i.e. the refection of f in the y −axis.

(d) Write down the equation of graph h, i.e. the refection of f in the x-axis.

(e) Write down the equation of the asymptote of g.

(f) Sketch the graphs of f, g and h on the same system of axes.

(g) Are f, g and h increasing or decreasing?

SOLUTION

(a) Domain: x∈R

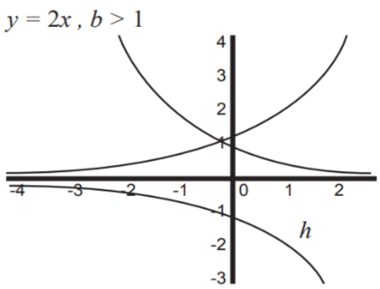
(b) Range: y > 0; y ∈ R

(c) $g(x) = 2^{-x}$ is the same as $g(x) = \left(\frac{1}{2}\right)^x$ which is a reflection of $f(x) = 2^x$ about the y-axis, since x has been replaced with – x.

(d) $h(x) = -f(x) = -2^x$

(e) y = 0

(f)



(h) f is an increasing function g is decreasing functions h is decreasing functions

8. INVERSE FUNCTIONS

8.1 STRAIGHT LINE

Example 1: Given: $f(x) = 2x - 4$

1.1 Determine f^{-1}, that is the inverse of f.

1.2 Sketch the graphs of f , $f^{-1}$and $y = x$ on the same system of axes.

Solution:

1.1 So if, $f(x) = 2x - 4$

f: $y = 2x - 4$

f^{-1}: $x = 2y - 4$ (interchange x and y)

$\therefore -2y = -x - 4$

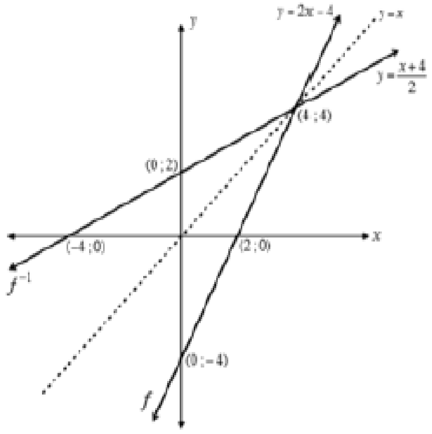
$\therefore 2y = x + 4$

$\therefore y = \frac{1}{2}x + 2$

We then say, inverse function of f is:

$f^{-1}(x) = \frac{1}{2}x + 2$

1.2



8.2 PARABOLA

Example 2: Given: $f(x) = 2x^2$

2.1 Determine f^{-1}, that is the inverse of f.

2.2 Sketch the graphs of f , $f^{-1}$and $y = x$ on the same system of axes.

Solution: 2.1

So if, $f(x) = x^2$

f: $y = x^2$

f^{-1}: $x = y^2$ (interchange x and y)

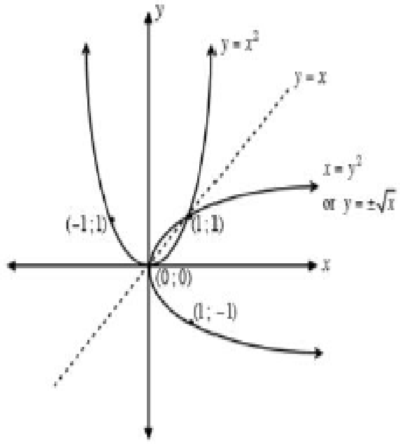
$\therefore y^2 = x$

$\therefore y = \pm\sqrt{x}$

We then say, inverse function of f is:

$f^{-1}(x) = \pm\sqrt{x}$

2.2



Example 3: (parabola with restricted domain).

Given: $f(x) = -\frac{1}{2}x^2 \;\; ; \; x \leq 0$

3.1 Determine f^{-1}, that is the inverse of f.

3.2 Sketch the graphs of f , f^{-1} and $y = x$ on the same system of axes.

Solution:

3.1

So, if, $f(x) = -\frac{1}{2}x^2 \quad x \leq 0$

f: $\quad y = -\frac{1}{2}x^2$

- interchange x and y

f^{-1}: $\quad x = -\frac{1}{2}y^2 \quad y \leq 0$

Choose only negative y

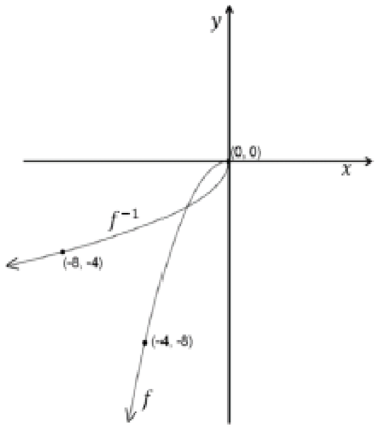
$\therefore y^2 = -2x$

$\therefore y = -\sqrt{-2x}$

We then say, inverse function of f is:

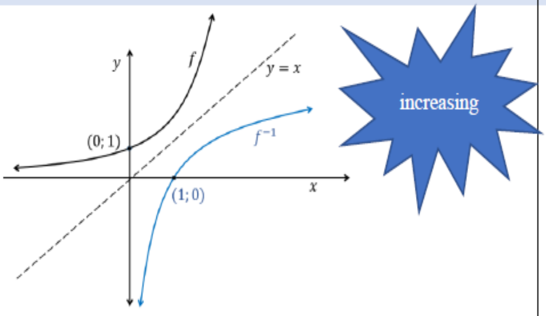
$f^{-1}(x) = -\sqrt{-2x}$

3.2

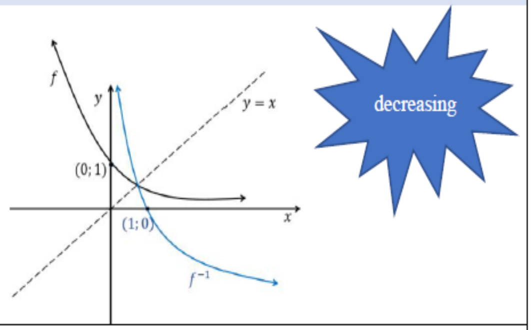


8.3 EXPONENTIAL AND LOGARITHMIC GRAPH	$f(x) = a^x$ and $f^{-1}(x) = log_a x$
The inverse of the exponential function $f(x) = a^x$ ($a > 0$; $a \neq 1$) is the logarithmic function $f^{-1}(x) = log_a x$.	

$a > 1$



$0 < a < 1$



Note:
- *The exponential graph f is an **increasing** graph* ($a > 1$)
- *The logarithmic graph f^{-1} is an **increasing*** ($a > 1$)

Note:
- *The exponential graph f is a **decreasing*** ($0 < a < 1$)
- *The logarithmic graph f is a **decreasing*** ($0 < a < 1$)

Domain and Range

f	*Domain:* $x \in R$	*Range:* $y > 0$
f^{-1}	*Range:* $y \in R$	*Domain:* $x > 0$

Inverse functions:

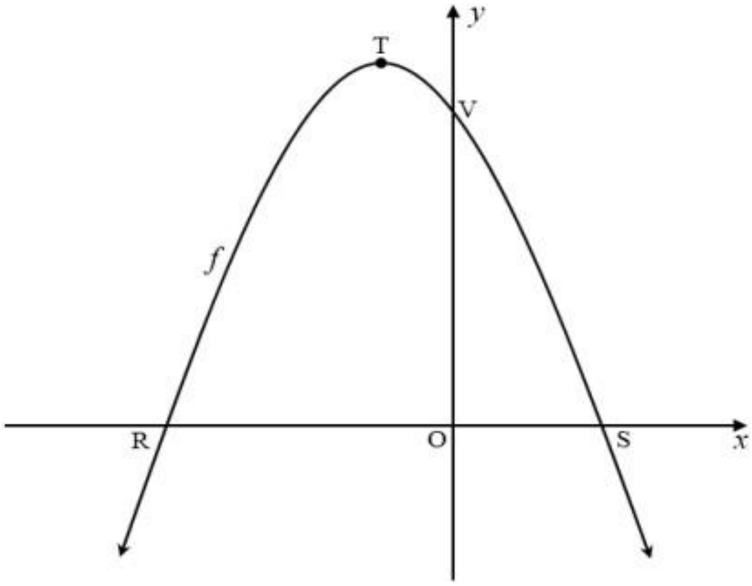
Graph f^{-1} is obtained by reflecting graph f about the line $y = x$. (the inverse of f)
- If $y = a^x$ then the inverse function is given by $x = a^y$ which can also be written as $y = log_a x$
- If $y = log_a x$ then the inverse function is given by $x = log_a y$ which can also be written as $y = a^x$

Asymptotes:
- *The exponential graph f has an **asymptote*** $y = 0$ ($x -$ **axis**)
- *The logarithmic graph f^{-1} has an **asymptote*** $x = 0$ ($y -$ **axis**)

Level 1 and 2 Activities

QUESTION 1

The diagram below shows the graph of $f(x) = -x^2 - 2x + 8$.
R and S are x-intercepts and V the y-intercept of f. T is the turning point of f.



1.1 Determine the length of RS. (4)

1.2 Determine the coordinates of T. (3)

1.3 The gradient of the tangent to the graph f at a point W is equal to 2.

1.3.1 Determine the coordinates of W. (4)

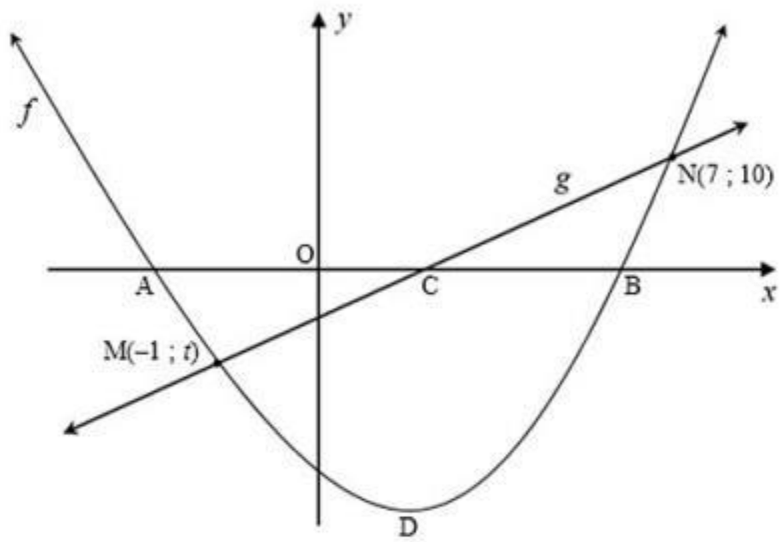
1.3.2 Determine the equation of a straight line, g , which is perpendicular to the tangent and passing through V. (2)

1.4 The graph of f is shifted one unit to the right and then reflected in the x-axis to produce a new function h. Determine the equation of h in the form: $h(x) = ax^2 + bx + c$. (4)

[17]

QUESTION 2

The diagram below shows the graphs of $f(x) = x^2 - 4x - 11$ and $g(x) = f'(x)$. A and B are the x-intercepts of f and C the x-intercept of g. D is the turning point of f. f and g intersect at M(–1 ; t) and N(7 ; 10).



2.1 Calculate the:

2.1.1 Coordinates of D (3)

2.1.2 Distance CN (4)

2.2 For which value(s) of x, is:

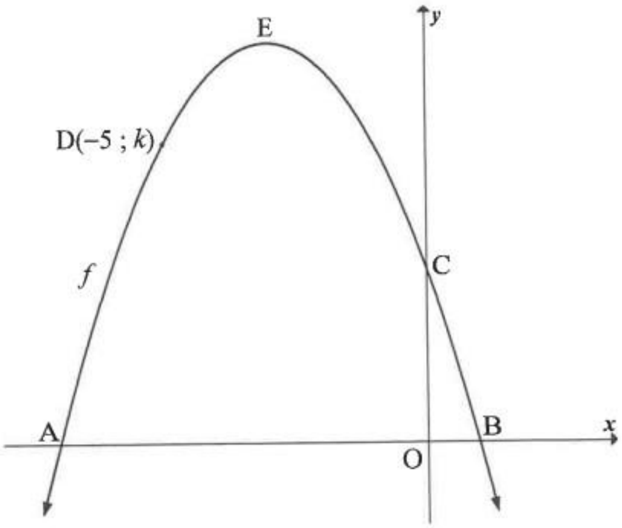
2.2.1 $f(x) < g(x)$? (2)

2.2.2 $g(x) - f(x)$ a maximum? (4)

[13]

QUESTION 3

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5\ ;\ k)$ is a point on f.



3.1 Calculate the coordinates of E, the turning point of f. (3)

3.2 Write down the value of k. (1)

3.3 Determine the equation of the straight line passing through C and D. (4)

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\ ;\ \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (2)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes and the turning point. (2)

QUESTION 5

The equation of a parabola is given by $f(x) = ax^2 + bx + c$. The roots of f are $(m-5)$ and $(m+3)$. The maximum value of f occurs at $x = 2$.

5.1 Determine the value of m. (2)

5.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$ if it is also given that $f(1) = 15$. (4)

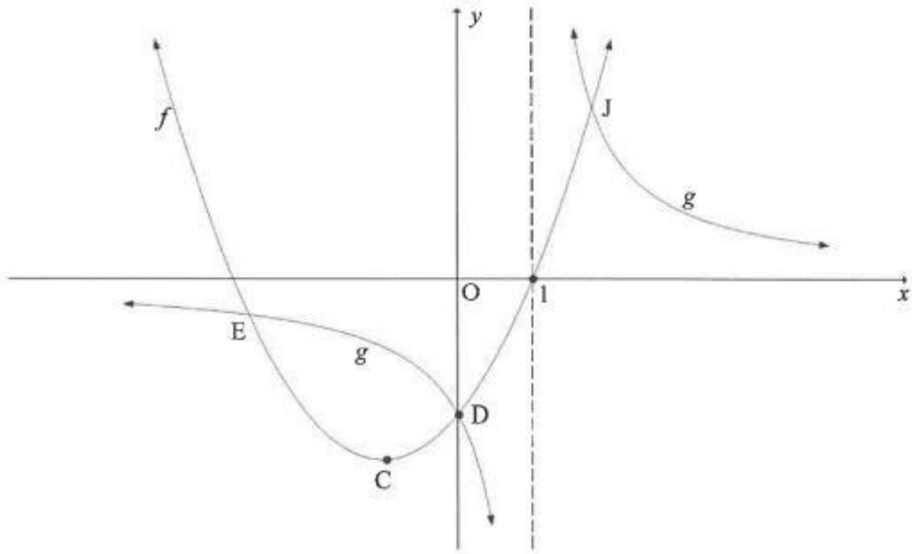
5.3 Determine the range of g if $g(x) = f(x) - 4$. (3)

[9]

QUESTION 6

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;\ 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



6.1.1 Write down the value of p. (1)

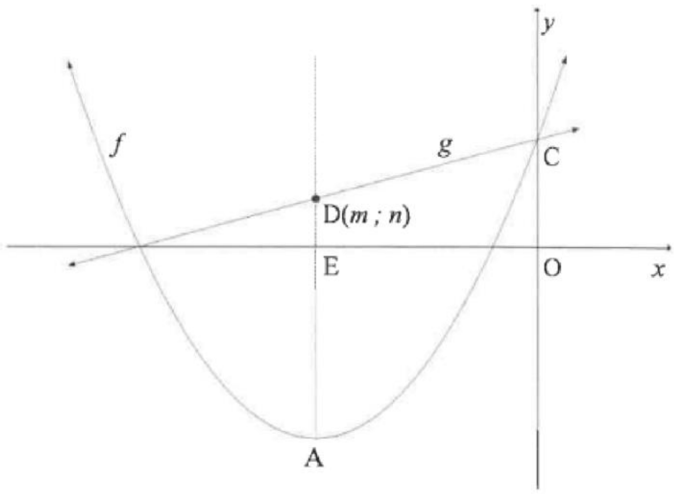
6.1.2 Show that $a = 3$ and $b = 2$. (3)

6.1.3 Calculate the coordinates of C. (4)

6.1.4 Write down the range of f. (2)

6.2 The graphs of $f(x)=\frac{1}{2}(x+5)^2-8$ and $g(x)=\frac{1}{2}x+\frac{9}{2}$ are sketched below.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at D(m ; n).
- C is the y-intercept of f and g.



6.2.1 Write down the coordinates of A. (2)

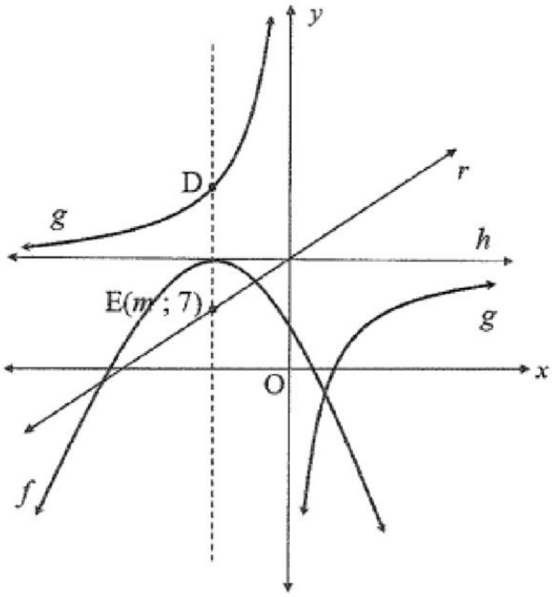
6.2.2 Write down the range of f. (1)

6.2.3 Calculate the values of m and n. (3)

QUESTION 7

Below are the graphs of $f(x) = -2(x+p)^2 + q$ and $g(x) = \frac{-3}{x} + n$.

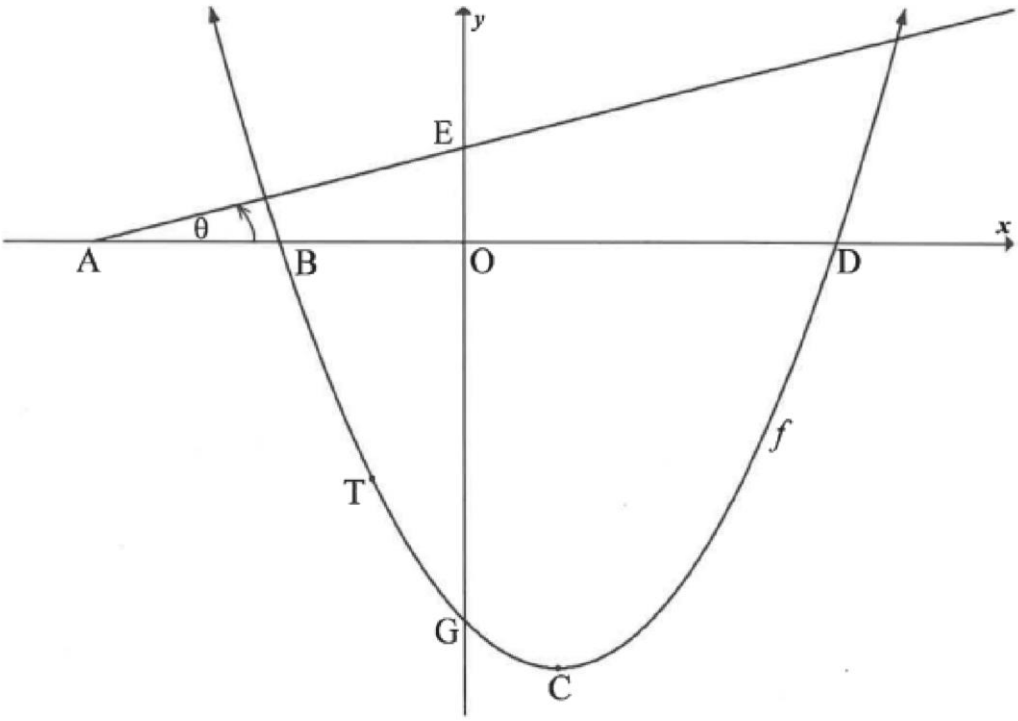
- $h(x) = n$, an asymptote of g, is also a tangent to f.
- The line $r(x) = x + 8$ is an axis of symmetry of g.
- $r(x) = x + 8$ also intersects the axis of symmetry of f in the point E(m ; 7).



7.1	Write down the domain of g.	(2)
7.2	Calculate the value of m.	(2)
7.3	Write down the value of n.	(1)
7.4	Given: $f(x) = -2(x+p)^2 + q$. Write down the values of p and q.	(2)
7.5	If it is given that $f(x) = -2x^2 - 4x + 6$, calculate the x-intercepts of f.	(3)
7.6	The axis of symmetry of f intersects the graph of g at point D. Determine the coordinates of D.	(2)

QUESTION 8

The graph of $f(x)=(x+4)(x-6)$ is drawn below.
The parabola cuts the x-axis at B and D and the y-axis at G.
C is the turning point of f.
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.
T is a point on f between B and G.



8.1 Write down the coordinates of B and D. (2)

8.2 Calculate the coordinates of C. (2)

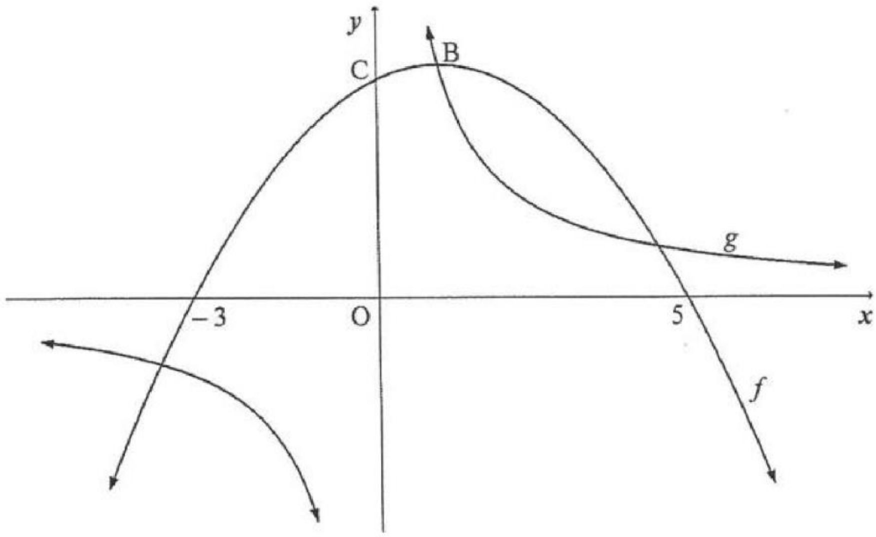
8.3 Write down the range of f. (1)

8.4 Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

8.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

QUESTION 9

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;\ 0)$ and $(5\ ;\ 0)$ and a y-intercept at C.



9.1 Write down the coordinates of the turning point of f. (2)

9.2 Calculate the coordinates of C. (2)

9.3 Calculate the value of d. (1)

9.4 Write down the range of g. (1)

QUESTION 10

Consider the graphs of $g(x)=\frac{6}{x+3}-\frac{3}{2}$ and $h(x)=\frac{6}{x-3}+2$.

10.1 Write down the domain of g. (1)

10.2 Write down the range of h. (1)

10.3 If the graph of g is shifted so that it coincides with the graph of h,

10.3.1 how many units must the graph be shifted horizontally? (1)

10.3.2 how many units must the graph be shifted vertically? (1)

Write down the equations of the asymptotes of g. (2)

10.4 Calculate the x-intercept of g. (1)

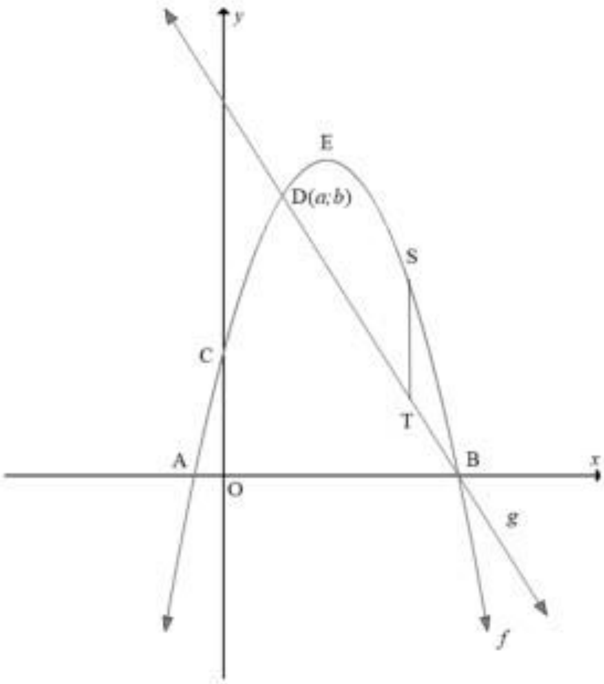
10.5 Sketch the graph of g in your ANSWER BOOK.
Show clearly all asymptotes and intercepts with the axes. (3)

QUESTION 11

The graphs of $f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ and $g(x) = -3x + 24$ are sketched below.

The graphs of f and g intersect at points D and B.

Points A and B are the x-intercepts of f.



11.1 Write down the coordinates of E, the turning point of f. (1)

11.2 Determine the average gradient of the curve of f between $x = 1$ and $x = 5$. (4)

11.3 Calculate the value of a, the x-coordinate of point D. (3)

QUESTION 12

Given: $f(x)=\frac{-1}{x-3}+2$

12.1 Write down the equations of the asymptotes of f. (2)

12.2 Write down the domain of f. (1)

12.3 Determine the coordinates of the x-intercept of f. (2)

12.4 Write down the coordinates of the y-intercept of f. (2)

12.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)

[10]

QUESTION 13

Given: $f(x)=\frac{8}{x-2}+2$

13.1 Write down the domain of f. (

13.2 Calculate the y-intercept of f. (

13.3 Calculate the x-intercept of f. (

13.4 Sketch the graph of f, clearly indicating the coordinates of the x and y-intercepts as well as the asymptotes. (

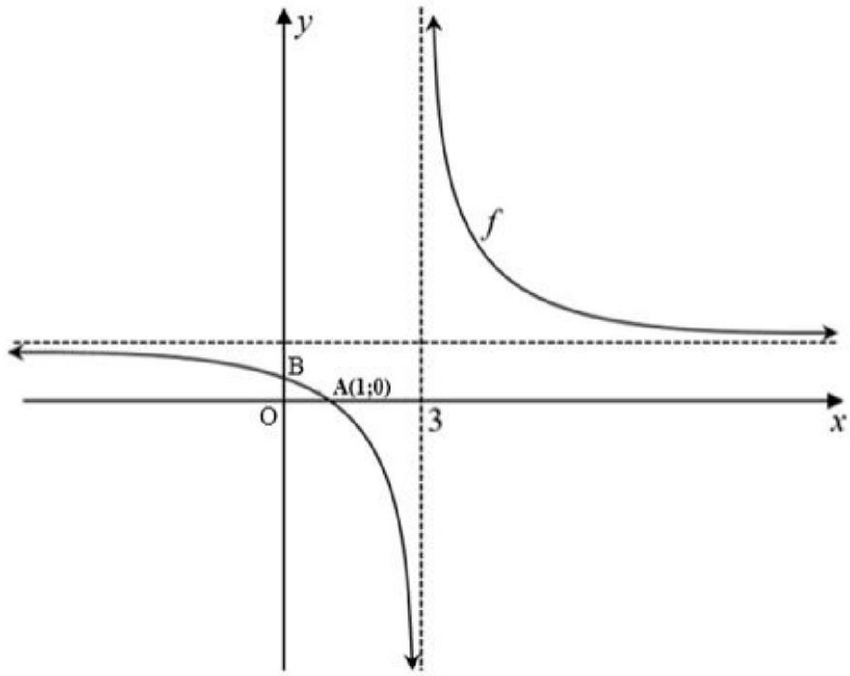
13.5 If $y=-x+k$ is an equation of the line of symmetry of f, determine the value of k. (

13.6 Determine the equation of the graph formed if f is shifted 3 units to the right and then reflected across the x-axis. (

[1

QUESTION 14

In the diagram below the graph of a hyperbolic function, $f(x)=\dfrac{x+k}{x+p}$, where k is a constant, is drawn. A(1 ; 0) and B are the x-intercept and y-intercept of f, respectively. The vertical asymptote goes through the x-axis at 3.



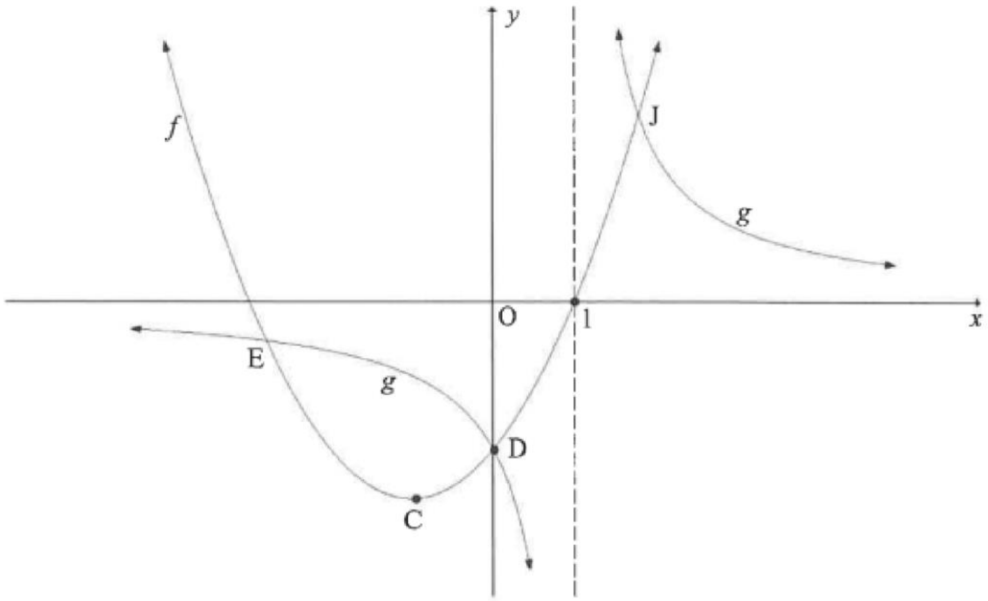
14.1	Write down the value of p.	(1)
14.2	Determine the value of k.	(2)
14.3	Calculate the coordinates of B.	(2)
14.4	Determine the values of x for which $x.f(x)\le 0$.	(3)
14.5	Rewrite the equation of f in the form $f(x)=\dfrac{a}{x+p}+q$.	(2)

[10]

QUESTION 15

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;\ 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



15.1 Write down the value of p. (

15.2 Show that $a = 3$ and $b = 2$. (

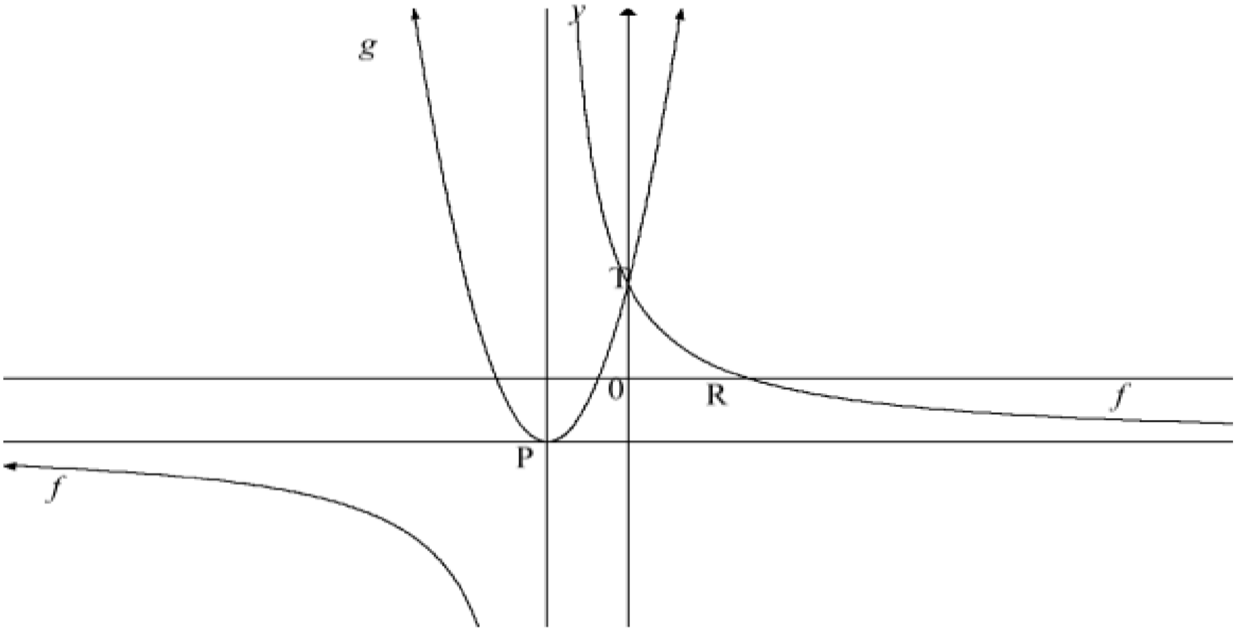
15.3 Calculate the coordinates of C. (

15.4 Write down the range of f. (

15.5 Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$ (

QUESTION 16

The diagram below shows the graphs of $f(x)=\frac{5}{x+p}+q$ and $g(x)=5x^2+10x+3$. The two graphs intersect at T, the y-intercepts of both graphs. R is the x-intercept of f. The asymptotes of f cut at P, the turning point of g.



16.1 Write down the coordinates of T. (2)

16.2 Calculate the coordinates of P. (4)

16.3 Write down the equation of f. (2)

16.4 Determine the equation of the:

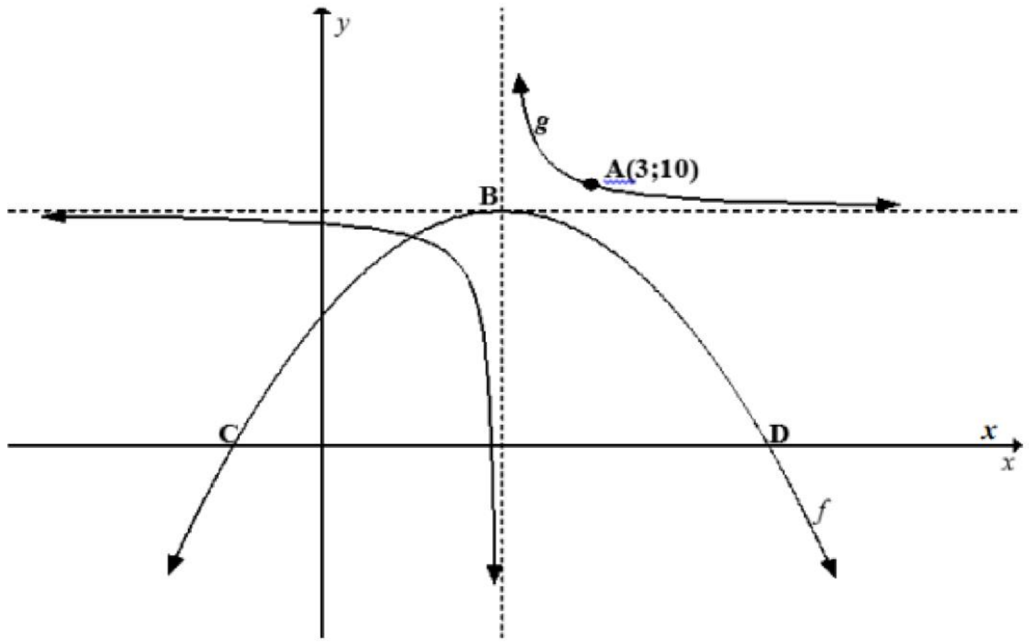
16.4.1 Tangent to g, touching f at T. (3)

16.4.2 Axis of symmetry of f, with a positive gradient. (3)

QUESTION 17

Sketched below are the graphs of $f(x)=-x^2+4x+5$ and $g(x)=\dfrac{1}{x+p}+q$. B is the turning point of f. The asymptotes of g intersect at B and the point A(3 ; 10) lies on g.

C and D are the x-intercepts of f.



17.1 Determine the coordinates of B. (3)

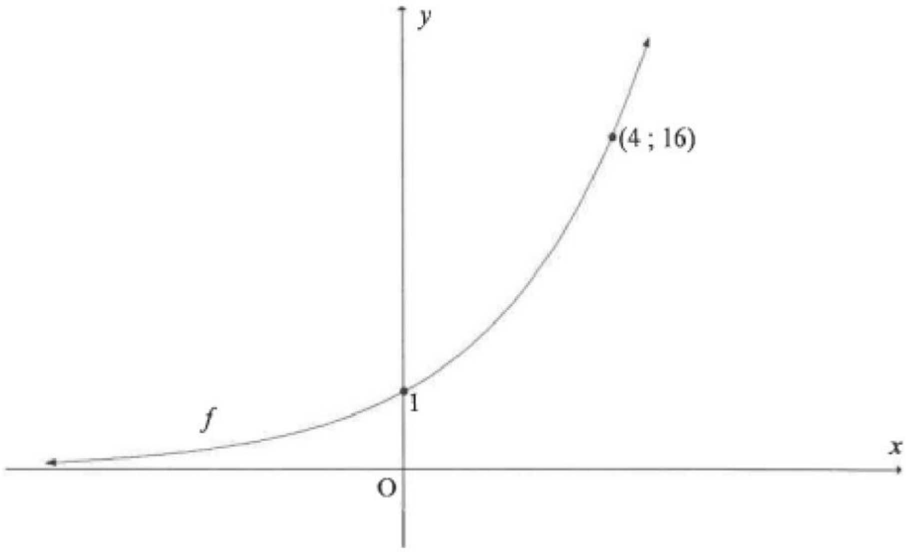
17.2 Hence, write down the values of p and q. (2)

17.3 Describe the nature of roots of the graph of t, if $t(x)=-f(x)+10$. (2)

17.4 The graph of h, where $h(x)=g(x+m)+n$ has asymptotes $x=4$ and $y=3$. Write down the value(s) of m and n. (2)

OUESTION 18

Sketched below is the graph of $f(x)=k^x\ ;\ k>0$. The point $(4\ ;\ 16)$ lies on f.

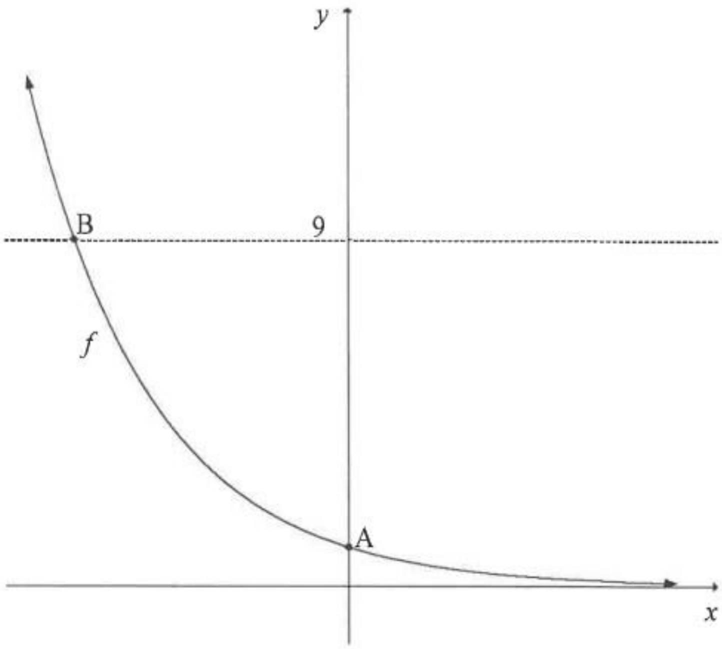


18.1 Determine the value of k. (2)

18.2 Graph g is obtained by reflecting graph f about the line $y=x$. Determine the equation of g in the form $y=\ldots$ (2) 1

QUESTION 19

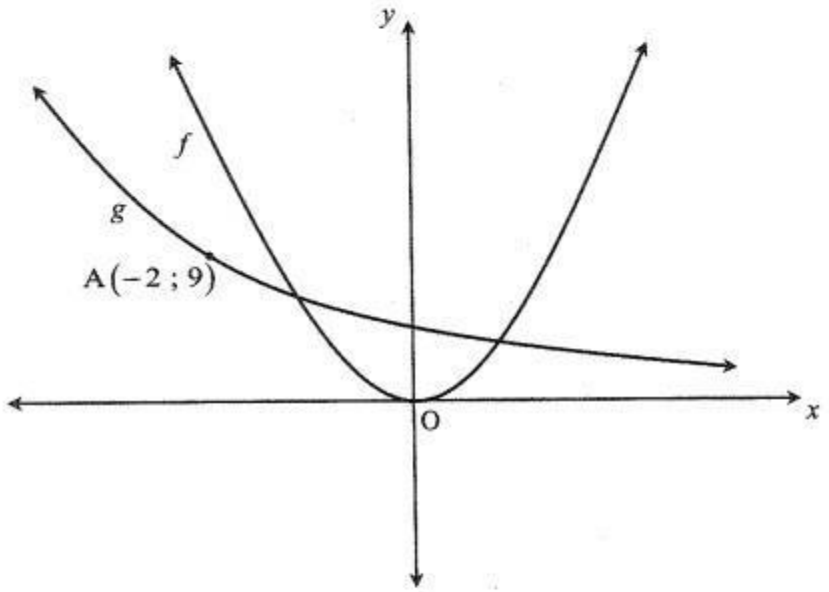
The graph of $f(x)=3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y=9$.



19.1	Write down the coordinates of A.	(1)
19.2	Determine the coordinates of B.	(3)
19.3	Write down the domain of f^{-1}.	(2)

QUESTION 20

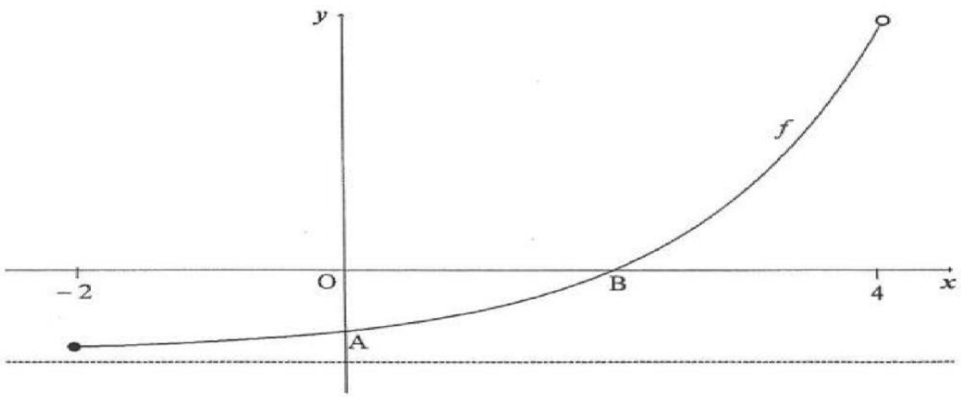
Sketched below are the graphs of $f(x) = 2x^2$ and $g(x) = \left(\frac{1}{3}\right)^x$. The point A$(-2\ ;\ 9)$ lies on the graph of g.



20.1 Determine the equation of $f^{-1}(x)$, the inverse of f, in the form $y = ...$ (3)

QUESTION 21

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2\,;4)$.
A and B are respectively the y- and x-intercepts of f.



21.1 Write down the equation of the asymptote of f. (1)

21.2 Determine the coordinates of B. (2)

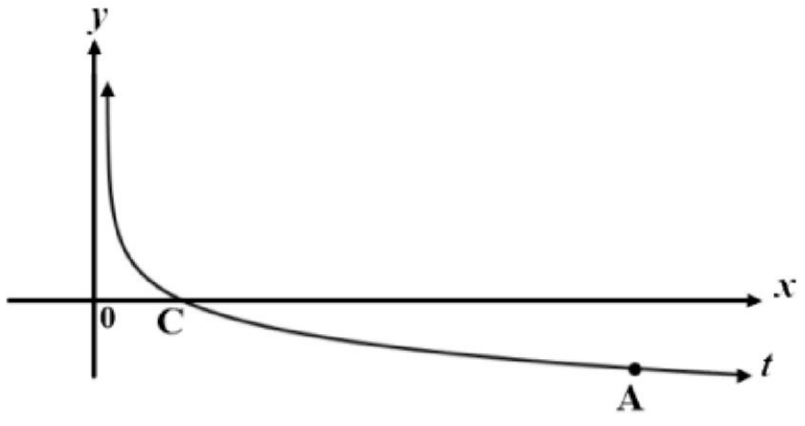
21.3 Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$ (3)

QUESTION 22.

The function defined by $t(x) = \log_a x$ is sketched below.

It is given that:

- Point C is the x-intercept of t.
- Point A (5; -1) is on t.



22.1 Write down the coordinates of point C. (1)

22.2 Write down the range of the graph of t. (1)

22.3 Calculate the value of a. (2)

QUESTION 23

Given the function: $f(x) = -3^x + 1$

23.1 Draw the graph of f in your ANSWER BOOK. Clearly show the intercepts with the axes as well as the asymptote of the graph. (3)

23.2 Write down the range of f. (2)

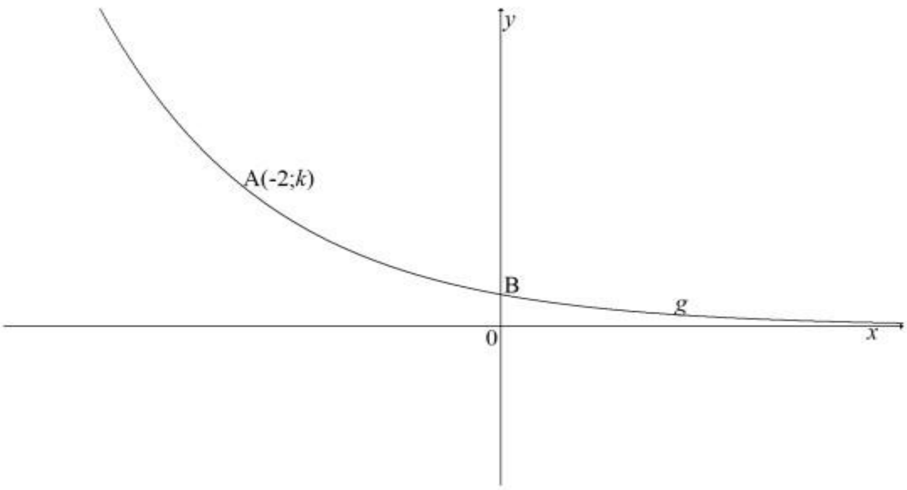
23.3 Determine the equation of the asymptote of g, given that $g(x) = -f(x)$. (2)

23.4 If g is shifted 1 unit upwards to give a new function h, determine the equation of h^{-1}, the inverse of h in the form $y = ...$ (3)

[10]

QUESTION 24

In the diagram, the graph of $g(x) = \left(\frac{1}{3}\right)^x$ is drawn. B is the y-intercept of g and $\mathrm{A}(-2\,;\,k)$ is another point on g.



24.1 Calculate the value of k. (2)

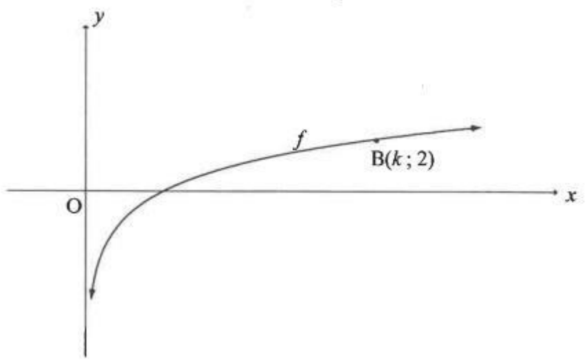
24.2 Determine the equation of g^{-1}, the inverse of g, in the form $y = ...$ (2)

24.3 Draw the graph of g^{-1} in your ANSWER BOOK. Clearly show the intercepts with the axes as well as the coordinates of any other point. (3)

QUESTION 25

The graph of $f(x)=\log_4 x$ is drawn below.

B(k ; 2) is a point on f.



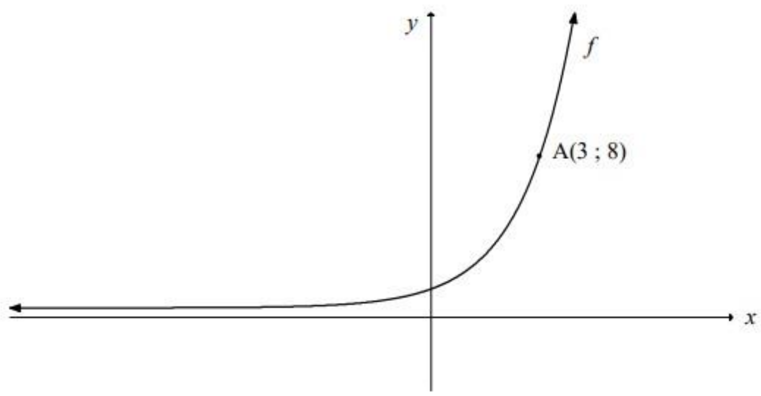
25.1 Calculate the value of k. (2)

25.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

25.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \dots$ (2)

QUESTION 26

In the diagram below, the graph of $f(x)=b^x$ is drawn. A(3 ; 8) is a point on f.



26.1 Calculate the value of b. (2)

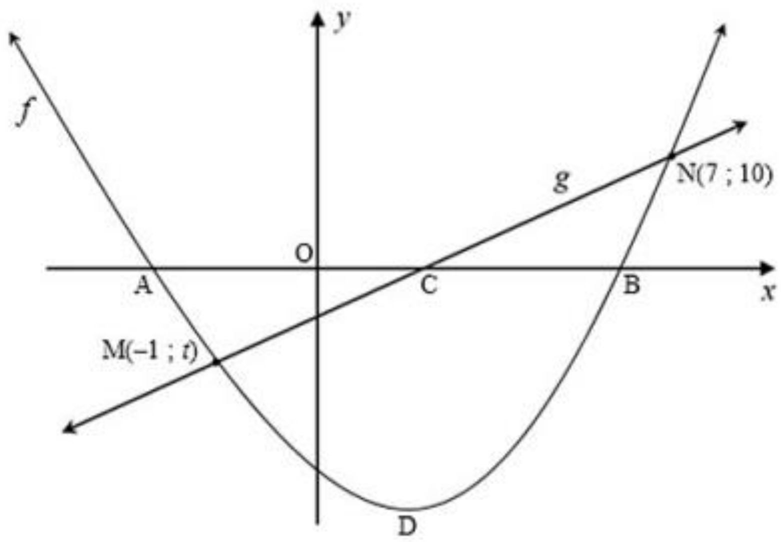
26.2 Determine the equation of f^{-1}, the inverse of f, in the form $y = \dots$ (2)

26.3 Sketch the graph of f^{-1}. Clearly show the intercept(s) with the axes, as well as the coordinates of ONE other point. (3)

PARABOLA AND STRAIGHT LINE (Levels 3 & 4 : 1.2)

QUESTION 1

The diagram below shows the graphs of $f(x) = x^2 - 4x - 11$ and $g(x) = f'(x)$. A and B are the x-intercepts of f and C the x-intercept of g. D is the turning point of f. f and g intersect at M$(-1\ ;\ t)$ and N$(7\ ;\ 10)$.



1.1 Calculate the:

1.1.1 Coordinates of D (3)

1.1.2 Distance CN (4)

1.2 For which value(s) of x, is:

1.2.1 $f(x) < g(x)$? (2)

1.2.2 $g(x) - f(x)$ a maximum? (4)

[13]

Question 2.5-2.7 are level 3 & 4

QUESTION 2

Given: $f(x) = -ax^2 + bx + 6$

2.1 The gradient of the tangent to the graph of f at the point $\left(-1\,;\,\frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

2.2 Calculate the x-intercepts of f. (2)

2.3 Calculate the coordinates of the turning point of f. (3)

2.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes and the turning point. (2)

2.5 Use the graph to determine the values of x for which $f(x) > 6$. (1)

2.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as the graph of f in QUESTION 2.4 Clearly indicate ALL intercepts with the axes. (2)

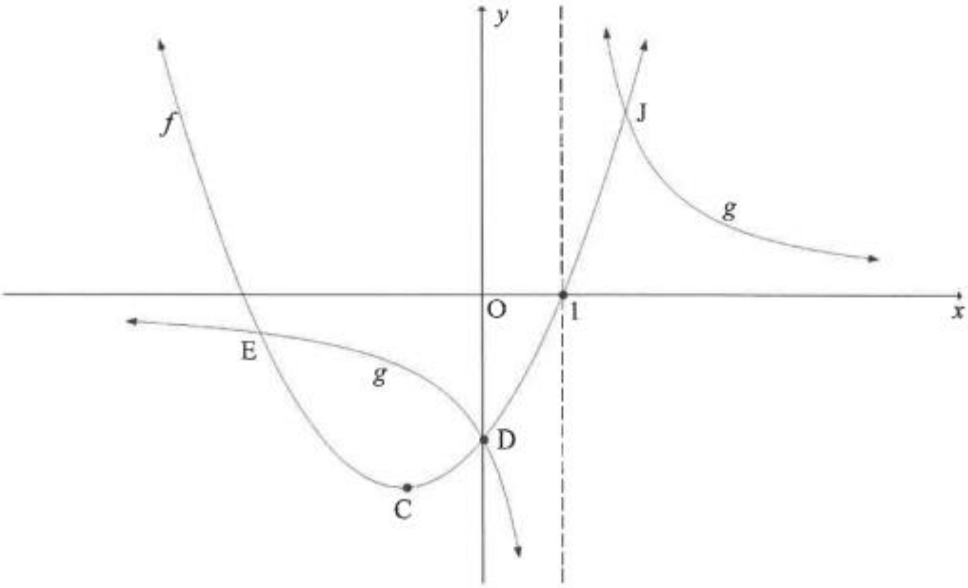
2.7 Write down the values of x for which $f(x).g(x) \leq 0$. (2)

[17]

QUESTION 3

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;\ 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



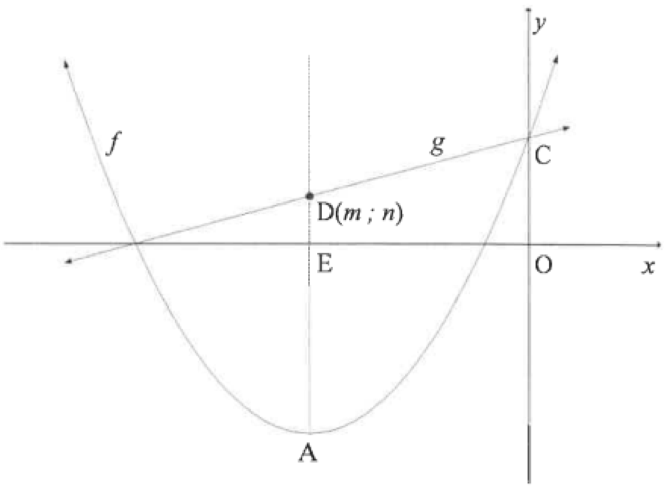
3.1.1	Write down the value of p.	(1)
3.1.2	Show that $a = 3$ and $b = 2$.	(3)
3.1.3	Calculate the coordinates of C.	(4)
3.1.4	Write down the range of f.	(2)
3.1.5	Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$	(3)
3.1.6	Is the straight line, determined in **QUESTION 3.1.5** a tangent to f? Explain your answer.	(2)
3.1.7	The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q.	(4)

[19]

3.2.4 – 3.2.6

3.2 The graphs of $f(x)=\frac{1}{2}(x+5)^2-8$ and $g(x)=\frac{1}{2}x+\frac{9}{2}$ are sketched below.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at D$(m\,;n)$.
- C is the y-intercept of f and g.



3.2.1 Write down the coordinates of A. (2)

3.2.2 Write down the range of f. (1)

3.2.3 Calculate the values of m and n. (3)

3.2.4 Calculate the area of OCDE. (3)

3.2.5 Determine the equation of g^{-1}, the inverse of g, in the form $y=\ldots$ (2)

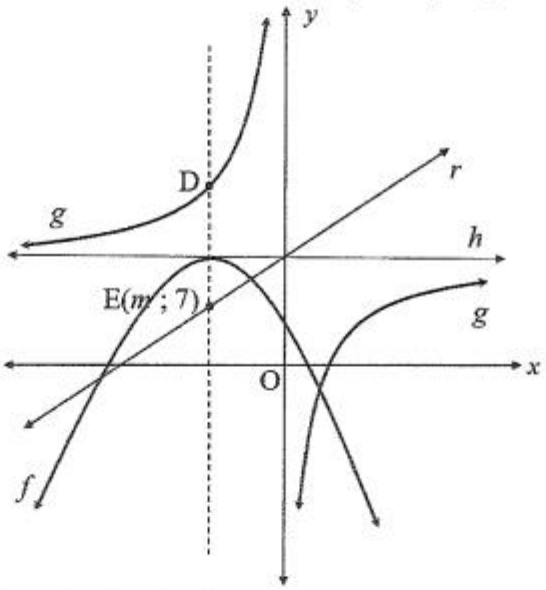
3.2.6 If $h(x)=g^{-1}(x)+k$ is a tangent to f, determine the coordinates of the point of contact between h and f. (4)

4.4 – 4.9

QUESTION 4

Below are the graphs of $f(x) = -2(x+p)^2 + q$ and $g(x) = \dfrac{-3}{x} + n$.

- $h(x) = n$, an asymptote of g, is also a tangent to f.
- The line $r(x) = x + 8$ is an axis of symmetry of g.
- $r(x) = x + 8$ also intersects the axis of symmetry of f in the point E(m ; 7).



4.1 Write down the domain of g. (2)

4.2 Calculate the value of m. (2)

4.3 Write down the value of n. (1)

4.4 Given: $f(x) = -2(x+p)^2 + q$. Write down the values of p and q. (2)

4.5 If it is given that $f(x) = -2x^2 - 4x + 6$, calculate the x-intercepts of f. (3)

4.6 The axis of symmetry of f intersects the graph of g at point D. Determine the coordinates of D. (2)

4.7 Determine the equation of the tangent to g at point D. Write down your answer in the form $y = bx + c$. (5)

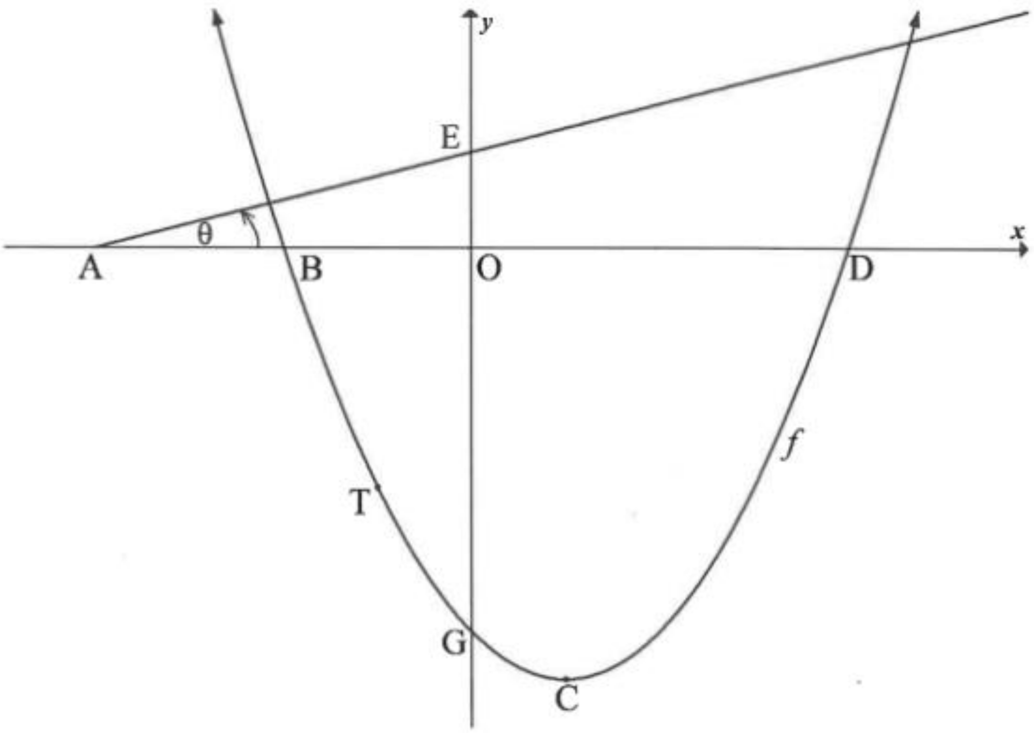
4.8 Determine the equation of $k(x)$ in the form $k(x) = \dfrac{a}{x+t} + s$ if k is the reflection of g about the line $x = 2$. (3)

4.9 Determine the value(s) of k for which the equation of $g(x+4) + k = 0$ will have a root that is less than -5. (3)

5.4.2 – 5.5

QUESTION 5

The graph of $f(x) = (x+4)(x-6)$ is drawn below.
The parabola cuts the x-axis at B and D and the y-axis at G.
C is the turning point of f.
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.
T is a point on f between B and G.

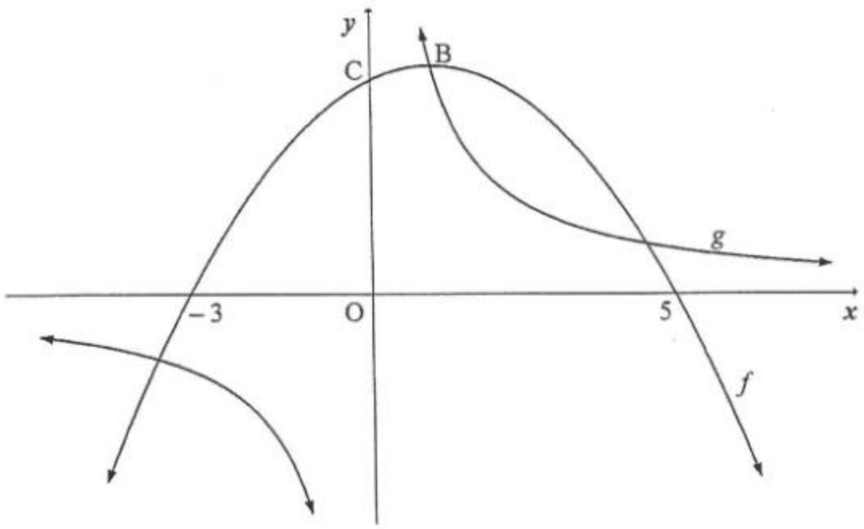


5.1	Write down the coordinates of B and D.	(2)
5.2	Calculate the coordinates of C.	(2)
5.3	Write down the range of f.	(1)
5.4	Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.	
5.4.1	Calculate the gradient of AE, correct to TWO decimal places.	(1)
5.4.2	Calculate the coordinates of T.	(5)
5.5	A straight line, g, parallel to AE, cuts f at K$(-3\,;-9)$ and R. Calculate the x-coordinate of R.	(6)

6.5 – 6.7

QUESTION 6

The graphs of $f(x)=-\frac{1}{2}(x-1)^2+8$ and $g(x)=\frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;0)$ and $(5\ ;0)$ and a y-intercept at C.



6.1	Write down the coordinates of the turning point of f.	(2)
6.2	Calculate the coordinates of C.	(2)
6.3	Calculate the value of d.	(1)
6.4	Write down the range of g.	(1)
6.5	For which values of x will $f(x).g(x)\leq 0$?	(3)
6.6	Calculate the values of k so that $h(x)=-2x+k$ will not intersect the graph of g.	(5)
6.7	h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y=f(x)+t$ intersects g at R.	(4)

[18]

7.7 - 7.9

QUESTION 7

Consider the graphs of $g(x)=\frac{6}{x+3}-\frac{3}{2}$ and $h(x)=\frac{6}{x-3}+2$.

7.1 Write down the domain of g. (1)

7.2 Write down the range of h. (1)

7.3 If the graph of g is shifted so that it coincides with the graph of h,

7.3.1 how many units must the graph be shifted horizontally? (1)

7.3.2 how many units must the graph be shifted vertically? (1)

7.4 Write down the equations of the asymptotes of g. (2)

7.5 Calculate the x-intercept of g. (1)

7.6 Sketch the graph of g in your ANSWER BOOK.
Show clearly all asymptotes and intercepts with the axes. (3)

7.7 Determine the value of k if $h(x)=-x+k$ is an axis of symmetry of g. (2)

7.8 For which value(s) of x will $\frac{6}{x+3}-\frac{3}{2}>-x+k$? (1)

7.9 The graph of g is reflected in the x-axis.
Write down the new equation in the form $y=\ldots$ (1)

[14]

8.5 – 8.7

QUESTION 8

Given: $f(x) = -ax^2 + bx + 6$

8.1 The gradient of the tangent to the graph of f at the point $\left(-1\ ;\ \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

8.2 Calculate the x-intercepts of f. (2)

8.3 Calculate the coordinates of the turning point of f. (3)

8.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes and the turning point. (2)

8.5 Use the graph to determine the values of x for which $f(x) > 6$. (1)

8.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as the graph of f in **QUESTION 8.5** Clearly indicate ALL intercepts with the axes. (2)

8.7 Write down the values of x for which $f(x).g(x) \leq 0$. (2)

[17]

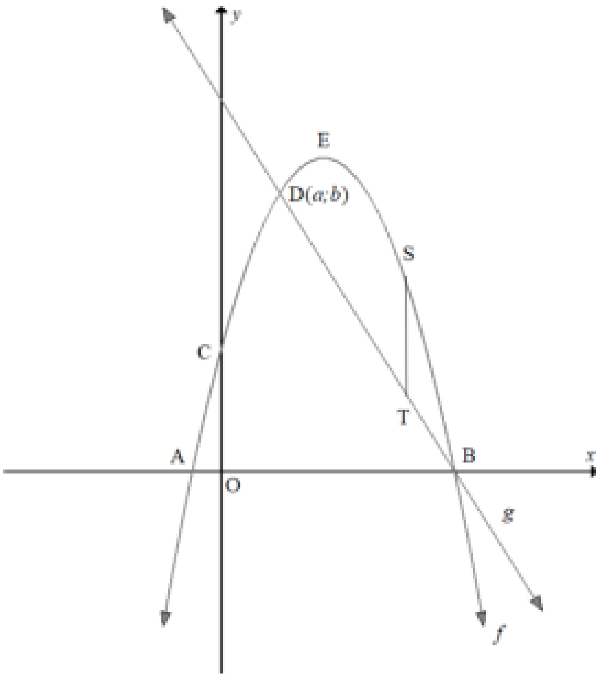
9.4 – 9.5

QUESTION 9

The graphs of $f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ and $g(x) = -3x + 24$ are sketched below.

The graphs of f and g intersect at points D and B.

Points A and B are the x-intercepts of f.



9.1	Write down the coordinates of E, the turning point of f.	(1)
9.2	Determine the average gradient of the curve of f between $x = 1$ and $x = 5$.	(4)
9.3	Calculate the value of a, the x-coordinate of point D.	(3)
9.4	Point S(x ; y) is a point on the graph of f, where $a \leq x \leq 8$. Line ST is drawn parallel to the y-axis with point T on the graph of g. Determine ST in terms of x.	(2)
9.5	Calculate the maximum length of ST.	(3) **[13]**

QUESTION 10.5 – 10.6

QUESTION 10

Given: $f(x) = \frac{8}{x-2} + 2$

10.1 Write down the domain of f. (2)

10.2 Calculate the y-intercept of f. (1)

10.3 Calculate the x-intercept of f. (2)

10.4 Sketch the graph of f, clearly indicating the coordinates of the x and y-intercepts as well as the asymptotes. (3)

10.5 If $y = -x + k$ is an equation of the line of symmetry of f, determine the value of k. (2)

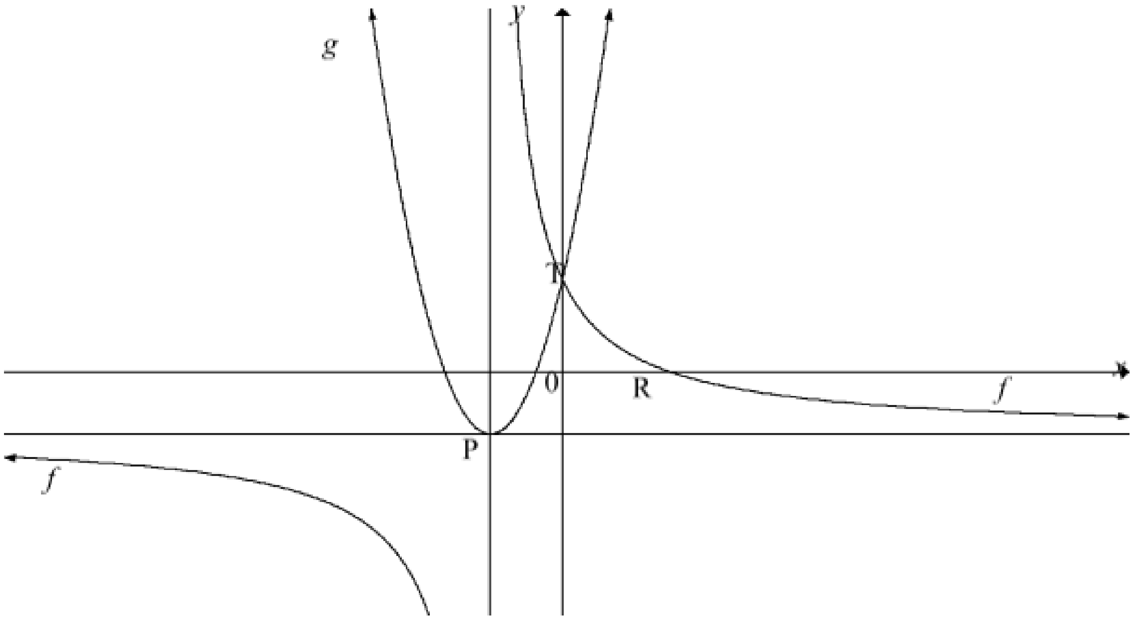
10.6 Determine the equation of the graph formed if f is shifted 3 units to the right and then reflected across the x-axis. (3)

[13]

11.4 and 11.5

QUESTION 11

The diagram below shows the graphs of $f(x)=\frac{5}{x+p}+q$ and $g(x)=5x^2+10x+3.$ The two graphs intersect at T, the y-intercepts of both graphs. R is the x-intercept of f. The asymptotes of f cut at P, the turning point of g.



11.1 Write down the coordinates of T. (2)

11.2 Calculate the coordinates of P. (4)

11.3 Write down the equation of f. (2)

11.4 Determine the equation of the:

11.4.1 Tangent to g, touching f at T. (3)

11.4.2 Axis of symmetry of f, with a positive gradient. (3)

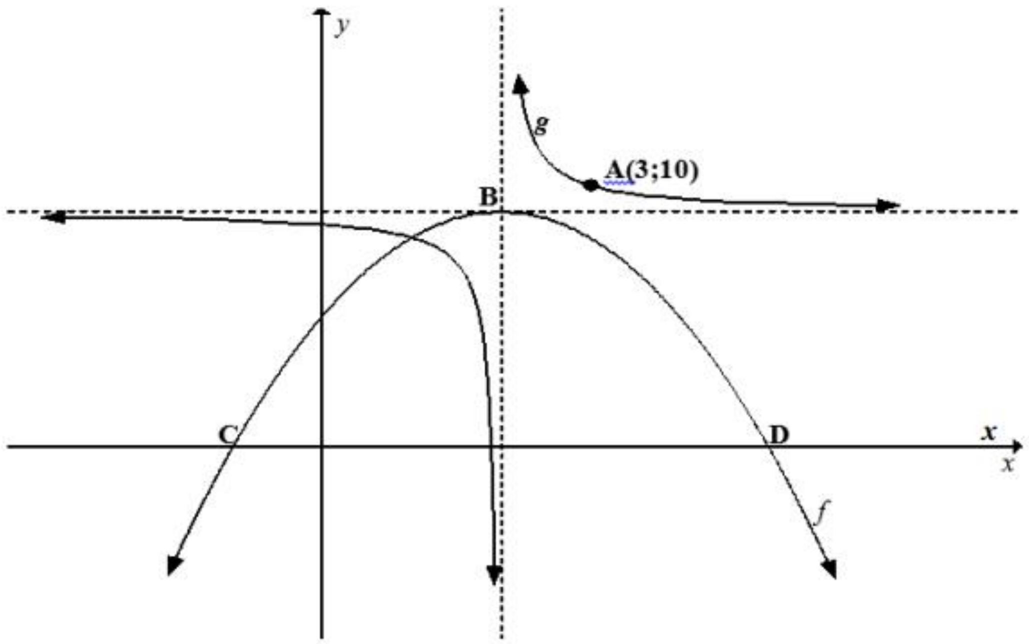
11.5 Determine for which values of x will $g^{/}(x)\times f(x)\leq 0$. (4)

[18]

QUESTION 12 (12. 5 – 12.6)

Sketched below are the graphs of $f(x) = -x^2 + 4x + 5$ and $g(x) = \dfrac{1}{x+p} + q$. B is the turning point of f. The asymptotes of g intersect at B and the point A(3 ; 10) lies on g.

C and D are the x-intercepts of f.



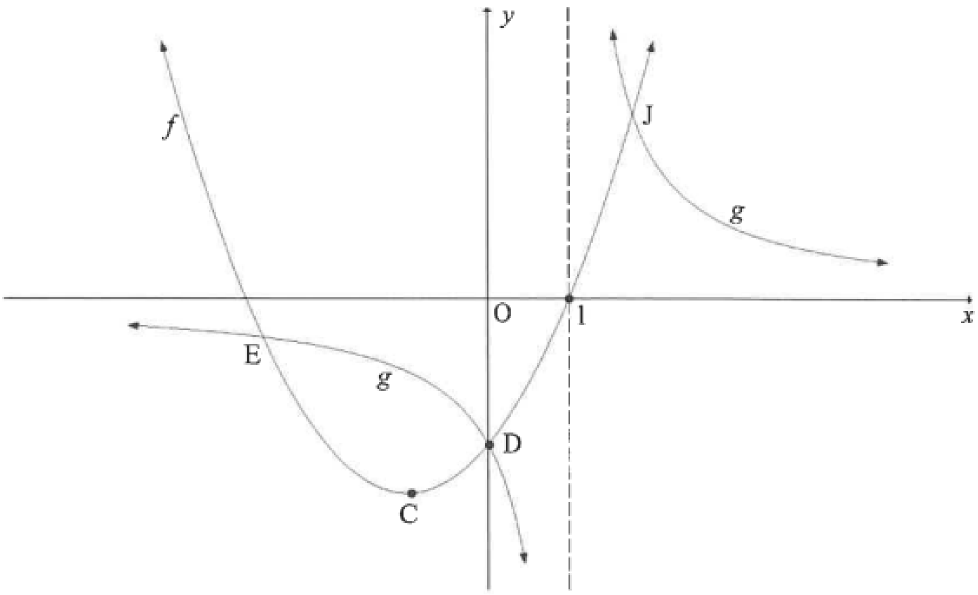
12.1	Determine the coordinates of B.	(3)
12.2	Hence, write down the values of p and q.	(2)
12.3	Describe the nature of roots of the graph of t, if $t(x) = -f(x) + 10$.	(2)
12.4	The graph of h, where $h(x) = g(x+m) + n$ has asymptotes $x = 4$ and $y = 3$. Write down the value(s) of m and n.	(2)
12.5	The tangent , $y = 8x + k$, touches the graph of f at P. Calculate the coordinates of P.	(4)
12.6	Determine the values of x for which:	
12.6.1	$g(x) \geq 10$	(2)
12.6.2	$f(x).g^{/}(x) > 0$	(4)

Question 13.3 -13.4
QUESTION 13.5-13.7

QUESTION 13 : 13. 5 – 13.7

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1 ; 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



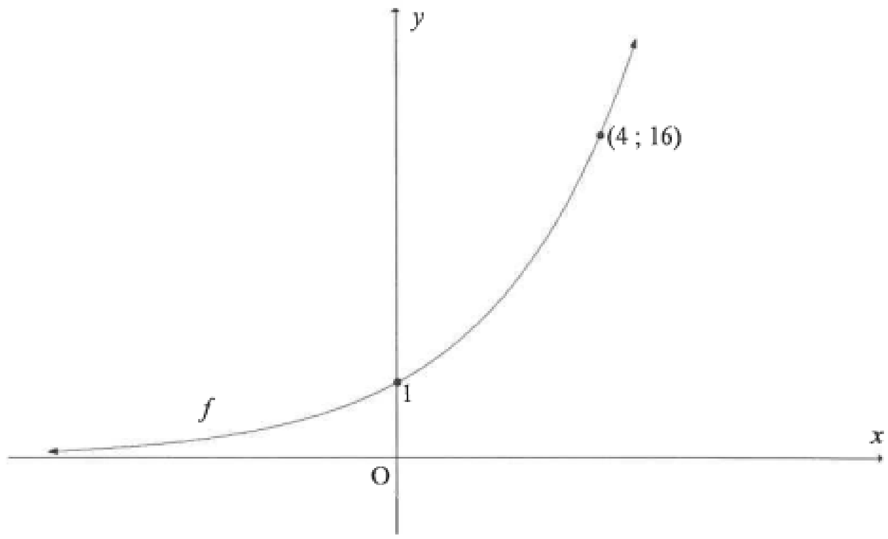
13.1	Write down the value of p.	(1)
13.2	Show that $a = 3$ and $b = 2$.	(3)
13.3	Calculate the coordinates of C.	(4)
13.4	Write down the range of f.	(2)
13.5	Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$	(3)
13.6	Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer.	(2)
13.7	The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q.	(4)

[19]

14.3 – 14.5

QUESTION 14

Sketched below is the graph of $f(x)=k^x$; $k>0$. The point $(4\ ;16)$ lies on f.



14.1 Determine the value of k. (2)

14.2 Graph g is obtained by reflecting graph f about the line $y=x$. Determine the equation of g in the form $y=\ldots$ (2)

14.3 Sketch the graph g. Indicate on your graph the coordinates of two points on g. (4)

14.4 Use your graph to determine the value(s) of x for which:

14.4.1 $f(x)\times g(x)>0$ (2)

14.4.2 $g(x)\leq -1$ (2)

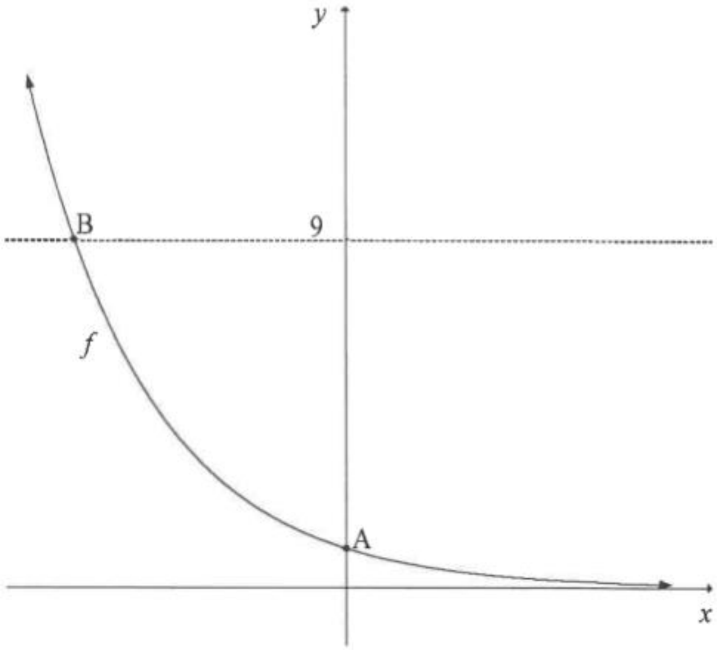
14.5 If $h(x)=f(-x)$, calculate the value of x for which $f(x)-h(x)=\dfrac{15}{4}$ (4)

[16]

15.4 – 15.5

QUESTION 15

The graph of $f(x)=3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y=9$.

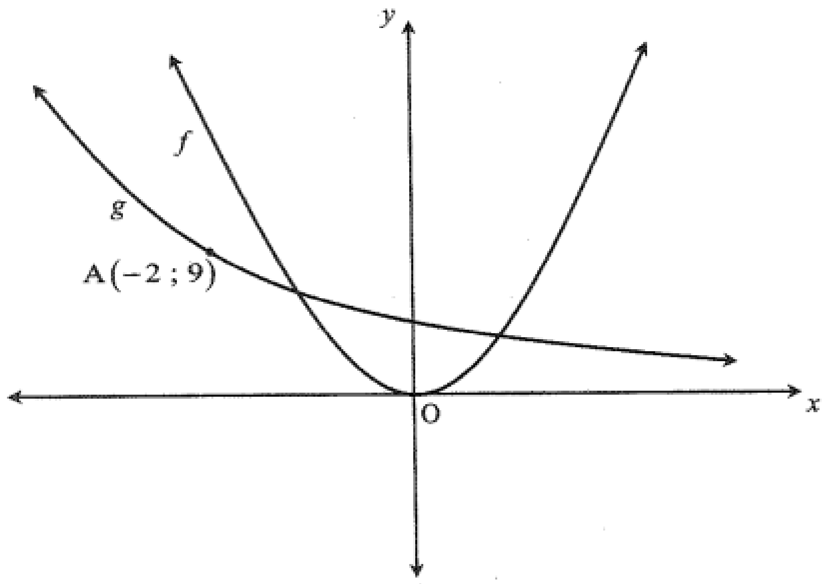


15.1	Write down the coordinates of A.	(1)
15.2	Determine the coordinates of B.	(3)
15.3	Write down the domain of f^{-1}.	(2)
15.4	Describe the translation from f to $h(x)=\dfrac{27}{3^x}$.	(3)
15.5	Determine the values of x for which $h(x)<1$.	(3)
		[12]

16.2 -16.3

QUESTION 16

Sketched below are the graphs of $f(x) = 2x^2$ and $g(x) = \left(\frac{1}{3}\right)^x$. The point $A(-2\ ;\ 9)$ lies on the graph of g.



16.1 Determine the equation of $f^{-1}(x)$, the inverse of f, in the form $y = ...$ (3)

16.2 For which values of x will $-2 \leq \log_{\frac{1}{3}} x \leq 0$. (3)

16.3 Simplify the following: $f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + \left[f^{-1}(x)\right]^2$. (3)

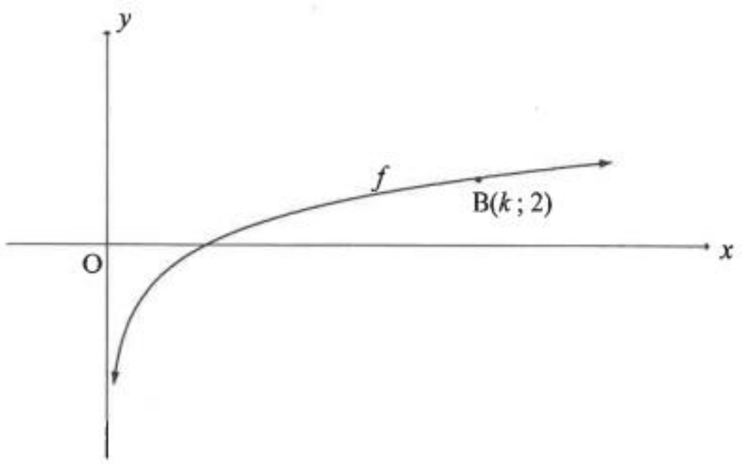
[9]

17.2 and 17.4

QUESTION 17

The graph of $f(x) = \log_4 x$ is drawn below.

B(k ; 2) is a point on f.



17.1 Calculate the value of k. (2)

17.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

17.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

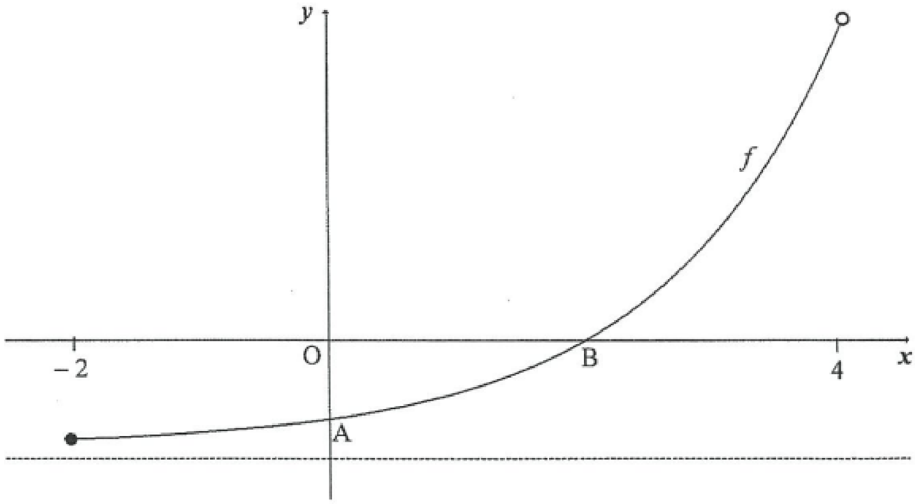
17.4 For which values of x will $x.f^{-1}(x) < 0$? (2)

[8]

18.4 – 18.7

QUESTION 18

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2\,;4)$.
A and B are respectively the y- and x-intercepts of f.

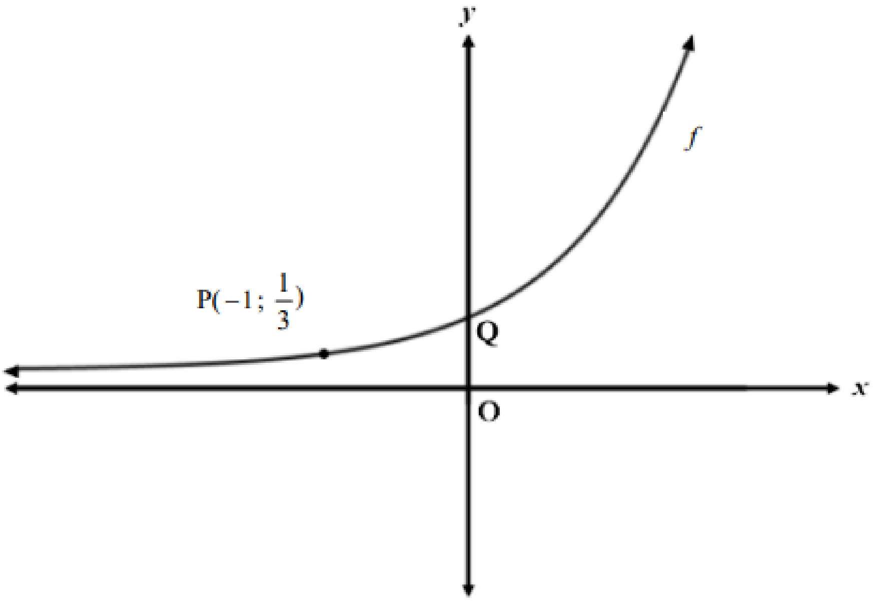


18.1	Write down the equation of the asymptote of f.	(1)
18.2	Determine the coordinates of B.	(2)
18.3	Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$	(3)
18.4	Calculate the vertical distance between k and f at $x = 1$	(3)
18.5	Write down the equation of g if it is given that $g(x) = f(x) + 4$	(1)
18.6	Write down the domain of g^{-1}.	(2)
18.7	Write down the equation of g^{-1} in the form $y = \ldots$	(2)
		[14]

19.1.3

QUESTION 19

The graph of $f(x) = 3^x$ is sketched below. $P(-1\,;\,\frac{1}{3})$ is a point on f.



19.1.1 Write f^{-1} in the form $y = \ldots$ (1)

19.1.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$ on the same set of axes in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes. (4)

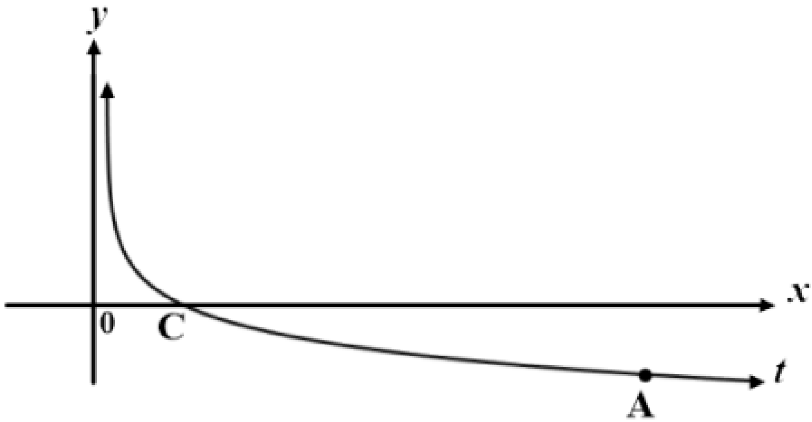
Use your graphs drawn in QUESTION 6.2 to solve for x if $\log_3(x-2) < 1$. (2)

[7]

19.2 The function defined by $t(x)=\log_a x$ is sketched below.

It is given that:

- Point C is the x-intercept of t.
- Point A (5; -1) is on t.



19.2.1 Write down the coordinates of point C. (1)

19.2.2 Write down the range of the graph of t. (1)

19.2.3 Calculate the value of a. (2)

19.3 The graph of $h=t(x)$ is transformed by a reflection of t about the line $y=x$.

19.3.1 Determine the equation of h in the form $y=$ … . (2)

19.3.2 Write down the domain of h. (1)

19.3.3 Write down the equation of the asymptote of h. (1)

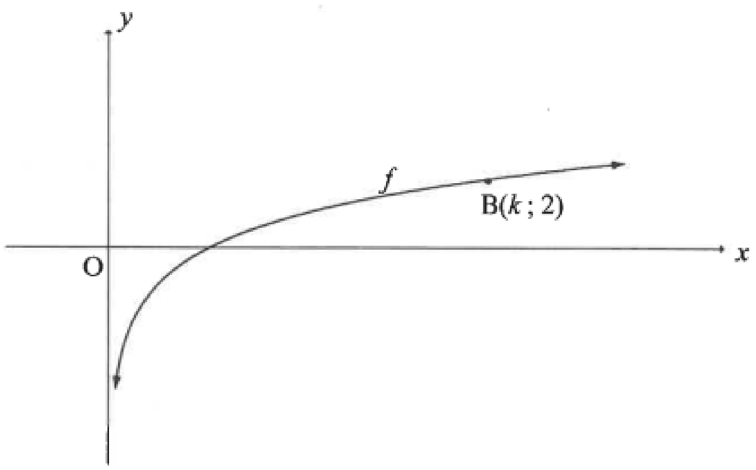
[16]

2

QUESTION 20

The graph of $f(x)=\log_4 x$ is drawn below.

B(k ; 2) is a point on f.



20.1 Calculate the value of k. (2)

20.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

20.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

20.4 For which values of x will $x.f^{-1}(x) < 0$? (2)

[8]

LEVEL 3 & 4, QUESTION 21.3-21.4

QUESTION 21

Given the function: $f(x)=-3^x+1$

21.1 Draw the graph of f in your ANSWER BOOK. Clearly show the intercepts with the axes as well as the asymptote of the graph. (3)

21.2 Write down the range of f. (2)

21.3 Determine the equation of the asymptote of g, given that $g(x)=-f(x)$. (2)

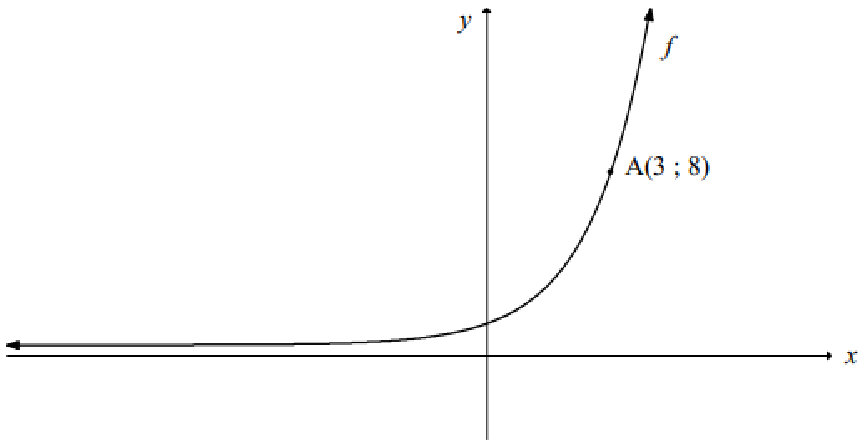
21.4 If g is shifted 1 unit upwards to give a new function h, determine the equation of h^{-1}, the inverse of h in the form $y = \ldots$ (3)

[10]

Question 22.4-22.5

QUESTION 22

In the diagram below, the graph of $f(x) = b^x$ is drawn. A(3 ; 8) is a point on *f*.



22.1	Calculate the value of *b*.	(2)
22.2	Determine the equation of f^{-1}, the inverse of *f*, in the form $y = \ldots$	(2)
22.3	Sketch the graph of f^{-1}. Clearly show the intercept(s) with the axes, as well as the coordinates of ONE other point.	(3)
22.4	Determine for which values of *x*, will $f^{-1}(x) < 4$.	(3)
22.5	Describe the transformation from *f* to $h(x) = \frac{1}{4}f(x)$.	(3)
		[13]

FINANCIAL MATHEMATICS MATERIAL DOCUMENT

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FINANCIAL MATHEMATICS (15±3) MARKS

1. EXAMINATION GUIDELINE

1.1 Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly/semi-annually.

1.2 With the exception of calculating i in the Fv and Pv formulae, candidates are expected to calculate the value of any of the other variables.

1.3 Pyramid schemes will NOT be examined in the examination

2. COMMON ERRORS AND MISCONCEPTIONS

(a) Many candidates fail to convert from one compounding period to another compounding period.
(b) Many candidates make an error of forgetting to divide the rate by compounding period given.
(c) Candidates fail to pick/choose the correct formula related to question asked.
(d) Some of the candidates round off during calculations, leading to wrong final answers.
(e) Many candidates fail to make "n" the subject of a formula.
(f) Most of the candidates do not know when to use present or future value formula.
(g) For future investments, most learners use wrong formula between compound interest and future value.
(h) Candidates commonly do not use their calculator correctly and they round off final answers incorrectly.

3. SUGGESTIONS FOR IMPROVEMENT

(a) Learners need deeper insight into the relevance of each of the formulae and under which circumstances each formula can be used. The variables in each formula must be explained. More practice in Financial Mathematics is necessary so that learners can identify when to use the different formulae.
(b) Teachers should demonstrate all the steps required when using a calculator. This should be done repetitively in class with every example done in Financial Mathematics. In formal assessment tasks at school, learners should be penalised for rounding off early.
(c) The difference between compound interest, future value- and present value annuities must be thoroughly explained.
(d) The correct Financial Mathematics language should be used in class and learners should read the question with understanding.
(e) Teachers need to emphasise – and learners need to practise – using different compounding periods for time intervals other than in years.

4. CONCEPTS TO BE COVERED IN FINANCIAL MATHEMATICS

4.1. Simple and compound interest

4.2. Grade11 compounding periods

4.3 Grade11 growth (appreciation) and decay (depreciation)

4.4 Time line

4.5 Grade11 nominal and effective interest rates

4.6 **New** annuities (future value and present Value (loan))

4.7 **New** sinking fund = appreciation – depreciation

4.8 **New** comparison of best loan and investment options.

FINANCIAL MATHEMATICS – Describe the application of mathematics and mathematical modelling to solve financial problems

5. FINANCIAL MATHEMATICS GRADE 11 SUMMARY NOTES

CALCULATE: $\boldsymbol{A, P, i \text{ and } n}$

5.1 SIMPLE INTEREST – It is where by the interest is calculated on the total amount invested or borrowed, and is not added onto the principal amount. Interest does not generate another interest.

Formulae

5.1.1 $\mathrm{A = P(1 + i.n)}$

5.1.2 $\mathrm{P = \frac{A}{(1+i.n)}}$

5.1.3 $n = \frac{\frac{A}{P}-1}{i}$

5.1.4 $i = \frac{\frac{A}{P}-1}{n}$

$\boldsymbol{A}$→ Future value/ accumulated amount/ final amount
$\boldsymbol{P}$ → Principal amount/ initial amount/ starting amount
$\boldsymbol{i}$ → Interest rate **(written as $\frac{i}{100}$)**
$\boldsymbol{n}$→ Period **(usually years)**

5.2 COMPOUND INTEREST- Allow interest to be earned on interest. Compound interest is advantageous for investing money but not for taking out a loan.

Formulae

5.2.1 $A = P(1 + i)^n$

5.2.2 $P = \frac{A}{(1+i)^n}$

5.2.3 $n = \log_{(1+i)} \frac{A}{P}$

5.2.4 $i = \sqrt[n]{\frac{A}{P}} - 1$

NB: n→ represents compounding period

5.3 COMPOUNDING PERIODS

Calculation of interest	i	n
ANNUALLY	i	n
SEMI-ANNUALY/ HALF-YEARLY	i÷2	n×2
MONTHLY	i÷12	n×12
QUARTERLY	i÷4	n×4
WEEKLY	i÷52	n×52
DAILY	i÷365	n×365

NB! Semi-annually/ half-yearly, monthly, and quarterly must be known for examination

6. GROWTH AND DECAY

- Once you bought expensive items they will lose value over a period of time. E.g. car or furniture loses its value as it becomes a scrap over a period of time as it being used. In Mathematics we called this is **DEPRECIATION**
- Other items gain value over a period of time. E.g. Property (house), house itself gain value as the home owner performs exterior and interior renovations that add to the price tag of the house. In Mathematics we called this is **APPRECIATION**
- That is why people are encouraged to invest on property rather than spend their money on buying expensive cars that will lose value over a period of time.

6.1 APPRECIATION (+)

Formulae

$A = P(1 + i)^n$ →compound appreciation

$A = P(1 + i.n)$ →simple appreciation

6.2 DEPRECIATION (–)

Formulae

$A = P(1 - i)^n$ → compound depreciation (reducing-balance or diminishing-method)

$A = P(1 - i.n)$ → simple depreciation (straight-line method)

NB: Book value - the depreciated value of an asset at any specified time

NB: Inflation – a broad rise in the prices of goods and services across the economy over time. Always use $\boldsymbol{A = P(1+i)^n}$ to calculate any related inflation question

7. NOMINAL INTEREST RATE AND EFFECTIVE INTEREST RATE

Effective interest rate- where the stated period and compounding period are the same.

It is where the compounding period is taken into consideration.

Nominal interest rate - where the stated period and compounding period are not the same.

$$\boxed{\left(1 + i_{eff}\right) = \left(1 + \frac{i_{nom}}{m}\right)^m}$$

$$\boxed{\left(1 + \frac{i_{new}}{n}\right)^n = \left(1 + \frac{i_{nom}}{m}\right)^m}$$

i_{eff} → effective interest rate

i_{nom} → Nominal interest rate

m → number of times interest receive a year **(compounding period)**

n → number of times interest receive a year **(compounding period)**

NB: In **GRADE 12** you have to know how to convert **NOMINAL INTEREST RATE** to **EFFECTIVE INTEREST RATE** or vice versa. Finally know how to CONVERT FROM ONE COMPOUNDING PERIOD TO ANOTHER

8. TIME LINE

- Where there are changes in amount (P) or interest rate (i) while invested.
- Calculations of more than one interests, deposits and withdrawals.
- Meaning that once there are more than one interest rates or deposits or withdrawals, we cannot use **SIMPLE/COMPOUND** interest directly. We use time line.
- Additional deposit positive sign (+) must be used
- Withdrawal negative sign (−) must be used

NB: Treat each amount separately and let it grow up to the last period.

NB: Round off only the final answer.

9. CONGNITIVE LEVELS 1&2 QUESTIONS

9.1 Class Activity/Home Activity A

1.1 A school buys tablets at a total cost of R140 000. If the average rate of

Inflation is 6.1% per annum over the next four years, determine the cost of replacing these tablets in 4 years' time. (3)

1.2 An investment earns interest at a rate of 7% per annum, compounded semi-annually. Calculate the effective annual interest rate on this investment. (3)

1.3 A car costing R198 000 has a book value of R102 755,34 after 3 years. If the value of the car depreciates at r% p.a. on a reducing balance, calculate r%. (5)

1.4 A machine costs R25 000 in 2016. Calculate the book value of the machine after 4 years if it depreciates at 9% p.a. according to the reducing balance method. (3)

1.5 The nominal interest rate of an investment is 12,35% p.a. compounded monthly. Calculate the effective interest rate. (4)

1.6 The value of the property increased from R145 000 to R221 292,32 over 6 years. Calculate the average annual rate of increase of the property over 6 years. (4)

9.2 Class Activity/Home Activity B

2.1 A company bought machinery costing R80 000, using reducing balance method, the machinery had a book value of R20 000 after 5 years Calculate the rate of depreciation. (3)

2.2 Calculate the effective interest rate if interest is compounded at 5% p.a., compounded quarterly. (3)

2.3 Calculate the effective interest rate per annum if an investment earns interest at a rate of 11.5% p.a., compounded monthly. (3)

2.4 Karabo bought a computer for R4 700. The value of the computer depreciated at a rate of 18% p.a. Using the reducing-balance method, calculate the book value of the computer 4 years after it was bought. (3)

9.3 Class Activity/Home Activity C

3.1 Lamola invests a lump sum of R5 000 in a saving account for exactly two years. The investment earns interest at 10% p.a., compounded quarterly.

3.1.1 what is the quarterly interest rate for Lamola's investment? (1)

3.1.2 calculate the amount in Lamola's savings account at the end of the 2 years. (3)

3.2 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (3)

9.4 Class Activity/Home Activity D

4.1 Given: $A = P(1 + in)$ where P and i are positive constants

4.1.1 State whether the graph of A, as a function of n, is linear or quadratic, exponential or none. (1)

4.1.2 Draw a possible graph of A, as a function of n. (2)

4.1.3 If n increases by 1, then determine the increase in A. (1)

4.2 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5)

4.3 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4)

10. ANNUITIES FLOW DIAGRAM

NEW GR12 FINANCIAL MATHEMATICS

1. INVESTMENT/FV

A)SINKING FUND

- ✓ Appreciation – gain value $A = P(1+i)^n$
- ✓ Depreciation – loose value $A = P(1-i)^n$
- ✓ **Sinking Fund** = Appreciation – depreciation
- ✓ **Monthly payment(x)** in sinking fund use FV formula.

B) ANNUITIES FUTURE VALUE

- Money **invested** on monthly bases to be used in future.
- Gain profit
- $FV = \dfrac{x[(1+i)^n - 1]}{i}$
- Calculation of i won't be asked in annuities.

2. LOAN TO BE PAID

A) ANNUITIES PRESENT VALUE

- Money borrowed to be paid back on monthly bases e.g Loan,NSFAS,
- Get money first and pay interest.
- $PV = \dfrac{x[1-(1+i)^{-n}]}{i}$
- Calculation of i won't be asked in annuities.
- **Outstanding balance**

NB! When you calculate n use log.

COMPARE BEST INVESTMENTS AND LOAN OPTIONS

- **Best investment option** - Whereby there is more **interest earned.**
- **Best Loan option** - Whereby there is less **interest paid.**

1. FUTURE VALUE ANNUITY

- **Future value** - is used in investments when you save money for future.

  E.g. **Saving account**, **Retirement fund** and **Sinking fund**.

- Regular payment (**usually monthly payments**) - it is like a present value that will collect interest over a period of time.

$$FV = \frac{x[(1+i)^n - 1]}{i}$$

Where F→ Future Value

x→ regular

i→ Interest rate

n→ Number of payments

HINTS ON FUTURE VALUE CALCULATIONS

- Calculate F, x and n → **Straight forward.** Won't be asked to calculate i
- **FUTURE VALUE - (There are 3 cases when calculating fv)**

3 cases

1. Payment made in **one month's time** → ***use formula as it is***
2. Payment starting at the **end of first month** → ***use formula as it is.***
3. Payment start **immediately** and **end on the last day** → (**include n+ 1** in the formula) $FV = \frac{x[(1+i)^n - 1]}{i}$

   - Since we have **immediately** and **end** at the same time it means one of the months it was **repeated**, meaning that payments were made twice. That is why we have **+1** for that additional payment made.

2. PRESENT VALUE ANNUITY

- **Present value** - used for loans e.g. Student loan (NSFAS), **Vehicle loan to buy cars, loan to buy house** (**Bond** or Mortgage or Home loan).
- **Present value** - Used when paying back the loan and interest is charged.
- First money **received** and **paid later.**
- **Present value (P)** - it is always the **outstanding balance** with **n** payments **to go**.

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

Where P→ Present Value
x→ Value of monthly instalment
i→ Interest rate
n→ Number of payments left / to be made

HINTS ON PRESENT VALUE CALCULATIONS

1. Calculate **P**, x and n **never** $\times$ **by 12** → **Straight forward.** Won't be asked to calculate i
2. Calculate OUTSTANDING BALANCE →**2** ***options***

   Option 1→Use **present value** formula but in **n**→ substitute number of payments left/ still **to go**.

   Option 2→ Use **future value** formula but **n**→ number of payments which have been **made**.

   Outstanding balance $= P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$ where P→***for loan***

2. Difficulties to pay e.g 103rd ,104th , 105th payments missed ,generate profit outside $A = P(1+i)^n$
   n = 3 since 3 payments missed. **1st calculate outstanding balance** for 102nd payment made
3. **FINAL PAYMENT**→(***you cannot calculate without outstanding balance***)
   → 1st **find outstanding balance**
   → 2nd use $P(1+i)^1$
   Where P→***outstanding balance***
   n =1 (since final payment will be made **1** month later)
4. **TOTAL INTEREST PAID** = Monthly payment×(n×compounding period) - **loan amount**

NB: **Compound interest formula** is only used when we have once off investments whereas **Future Value formula** is used when we have compounding periods (multiple instalments)

3. SINKING FUND

- **SINKING FUND**- It is a **saving account** which is set up in order to save money to **replace an old** item in future. It is used as a saving account that will allow investor to purchase/buy expensive items or to fund expensive capital outlays in few years time
- **Calculate** the REPLACEMENT/EXPECTED/NEW COST →(**APPRECIATION**)
- **Calculate** the SCRAP VALUE/BOOK/TRADE-IN/DECAY VALUE →(**DEPRECIATION**)

SINKING FUND = APPRECIATION - DEPRECIATION

✓ To calculate **monthly instalment** in the **sinking fund** →Use the <u>**future value**</u> annuity formula

11.COGNITIVE LEVEL 1&2 (ANNUITIES)

11.1 Class Activity/Home Activity A

1.1 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

1.1.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

1.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14%p.a. compounded monthly. The first payment was made at the end of the first month.

1.2.1 Show that the loan would be paid off in 234 months. (4)

1.3 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

11.2 Class Activity/Home Activity B

Sebe bought a house. He paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. He obtained a loan from the bank to pay the balance of the selling price. The bank charges him interest of 9% per annum, compounded monthly.

2.1 Determine the selling price of the house. (1)

2.2 The period of the loan is 20 years and he starts repaying the loan one month after it was granted. Calculate his monthly instalment. (4)

2.3 Calculate the balance of his loan immediately after 85^{th} instalment. (3)

11.3 Class Activity/Home Activity C

3.1 Exactly five years ago Maleka bought a car for R145 000. The current book value of this car is R72 500. If the car depreciates by fixed annual rate according to reducing-balance method, calculate the rate of depreciation. (3)

3.2 Malodi took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one months after the loan is granted.

3.2.1 Calculate the value of Malodi's monthly instalment. (4)

3.2.2 Mphekgoane took out a loan for the same amount and at the same interest rate as Malodi. Mphekgoane decides to pay R6 000 at the end of every month. Calculate how many months it took for Mphekgoane to settle the loan. (4)

3.2.3 Who pays more interest, Malodi or Mphekgoane? Justify your answer. (2)

11.4 Class Activity/Home Activity D

4.1 Modiba started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, he deposited R400 into an annuity fund. He continued doing this until he retired on 31 December 2013.

4.1.1 Determine the total amount of money that he paid into the fund. (2)

4.1.2 The interest rate on this fund was 8% p.a., compounded monthly.

Calculate the value of the fund at the time that he retired. (5)

4.1.3 On 1 January 2014 Modiba invested R2 million in account paying interest at 10% p.a., compounded monthly. Modiba withdraws a fixed amount from this account at the end of each month, starting on the 31 January 2014. If Modiba wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount he could withdraw at the end of each month. (4)

11.5 Class Activity/Home Activity E

5.1 Galane invested R10 000 for 3 years at an interest rate of r% p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

5.2 Refiloe takes a loan from a bank to buy a car for R235 000. She agrees to repay month after the loan is granted. The bank charges interest at 11% p.a, compounded monthly.

5.2.1 Calculate Refiloe's monthly instalment. (4)

5.2.2 Calculate the total amount of interest that Refiloe will pay during the first year of the repayment of the loan. (6)

12. COGNITIVE LEVEL 1&2 SOLUTIONS (GRADE 11 REVISION)

12.1 CLASS ACTIVITY/HOME ACTIVITY A

1.1

12.2

8.1	$A = P(1+i)^n$ $= 140\ 000(1+0{,}061)^4$ $= R177\ 414{,}69$	✓140 000 ✓$(1+0{,}061)^4$ ✓answer/antwoord (3)

1.1	$A = P(1-i)^n$ $20000 = 80000(1-i)^5$ $0{,}25 = (1-i)^5$ $\sqrt[5]{0{,}25} = 1-i$ $i = 1-\sqrt[5]{0{,}25}$ $i = 0{,}24214417$ $i = 24{,}21\%$	✓substitution into correct formula/ *verv.in korrekte vorm* ✓simplification/*vereenv* ✓answer/*antw.* (3)
1.2	$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$ $1+i_{eff} = \left(1+\frac{0{,}05}{4}\right)^4$ $i_{eff} = 0{,}050945336...$ Effective rate = 5,09 % p.a.	✓vorm/*vorm* ✓subst/*verv* ✓answer/*antw.* (3)
1.	$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$ $1+i_{eff} = \left(1+\frac{0{,}115}{12}\right)^{12}$ $i_{eff} = \left(1+\frac{0{,}115}{12}\right)^{12} - 1$ $i_{eff} = 12{,}13\%$	✓formula/*form.* ✓$i = \frac{0{,}115}{12}$ ✓answer/*antw.* (3)
1.4	$A = P(1-i)^n$ $= 4\ 700(1-0{,}18)^4$ $= R\ 2124{,}97$	✓formula/*form.* ✓substitution/*verv.* ✓answer/*antw.* (3)

1.5

1.6 / 8.3	$A = P(1+i)^n$ $R221\ 292{,}32 = R145\ 000\left(1+\frac{r}{100}\right)^6$ $\sqrt[6]{\frac{R\ 221\ 292{,}32}{145\ 000}} = 1+\frac{r}{100}$ $\frac{r}{100} = 0{,}07300000324$ $r = 7{,}3\%$	✓correct substitution into correct formula ✓$n = 6$ ✓ $\sqrt[6]{\frac{R\ 221\ 292{,}32}{145\ 000}} = 1+\frac{r}{100}$ ✓ answer/*antw.* (4)

ACTIVITY/HOME ACTIVITY B

12.3 CLASS ACTIVITY/HOME ACTIVITY C

3.1.	Quarterly interest rate/*Kwartaallikse rentekoers* $=\dfrac{10\%}{4}$ $=2{,}5\%$	✓answer (1)
3.2.	$A=P(1+i)^n$ $=5000\left(1+\dfrac{2{,}5}{100}\right)^{2\times 4}$ $=R6\,092{,}01$	✓$n=8$ ✓ $5000\left(1+\dfrac{2{,}5}{100}\right)^{2\times 4}$ ✓answer (3)
3.3	$A=P(1+i)^n$ $23000=1570(1.12)^n$ $(1.12)^n=14{,}64968153..$ $n\log(1{,}12)=\log 14{,}64968153..$ $n=23{,}69$ years (23,68701...) or $n=24$ years or $n=23$ years 8 months or $n=23{,}7$ years **OR** $A=P(1+i)^n$ $23000=1570(1+\dfrac{12}{100})^n$ $(1.12)^n=14{,}64968153..$ $n\log(1{,}12)=\log 14{,}64968153..$ $n=23{,}69$ years (23,68701...) or $n=24$ years or $n=23$ years 8 months or $n=23{,}7$ years Note: Accept 24 years : 4 / 4 Incorrect Formula: 0/4	✓ formula ✓ substitution ✓ apply log function ✓answer (4) ✓ formula ✓ substitution of $\dfrac{12}{100}$ ✓ apply log function ✓answer (4)

12.4 CLASS ACTIVITY/HOME ACTIVITY D

1.2 1.1.	$A = P(1+i)^n$ $2P = P\left(1+\frac{r}{4}\right)^{6\times 4}$ $2 = \left(1+\frac{r}{4}\right)^{24}$ $1+\frac{r}{4} = 2^{\frac{1}{24}}$ $r = 4\left(2^{\frac{1}{24}} - 1\right)$ $r = 4\left(2^{\frac{1}{24}}\right) - 4$ $r = 0,1172\ \ldots$ rate = 11,72% p.a. compounded quarterly **Note:** Penalty 1 for incorrect rounding off. **OR** $A = P(1+i)^n$ $2P = P\left(1+\frac{r}{400}\right)^{6\times 4}$ $2 = \left(1+\frac{r}{400}\right)^{24}$ $1+\frac{r}{400} = 2^{\frac{1}{24}}$ $r = 400\left(2^{\frac{1}{24}} - 1\right)$ $r = 400\left(2^{\frac{1}{24}}\right) - 400$ $r = 11,72\%$ p.a.	✓ 2P ✓ $\frac{r}{4}$ and 24 ✓ $1+\frac{r}{4} = 2^{\frac{1}{24}}$ ✓ $r = 4\left(2^{\frac{1}{24}}\right) - 4$ ✓ answer (5) ✓ 2P ✓ $\frac{r}{400}$ and 24 ✓ $1+\frac{r}{400} = 2^{\frac{1}{24}}$ ✓ $r = 400\left(2^{\frac{1}{24}}\right) - 400$ ✓ answer (5)	(1) (2)
1.	$A = P(1+i)^n$ $1711,41 = 1430,77\left(1+\frac{i}{12}\right)^{18}$ $\left(1+\frac{i}{12}\right)^{18} = 1,196146131\ldots$ $1+\frac{i}{12} = 1,009999937\ldots$ $i = 0,1199992431\ldots$ Rate = 12, 00% p.a. compounded monthly. **OR** $\left[\frac{1711,41}{1430,77}\right]^{\frac{1}{18}}$ $= 1,00999\ldots$ $\therefore i = 12(1,01-1)$ $= 0,12$ $= 12\%$	✓ substitution ✓ $\left(1+\frac{i}{12}\right)^{18} = 1,196146131\ldots$ ✓ $1+\frac{i}{12} = 1,009999937\ldots$ ✓ answer (4)	(1)
	OR $A(1) - A(0) = (P+Pi) - (P+0) = Pi$		

13. COGNITIVE LEVEL 1&2 SOLUTIONS (ANNUITIES)

13.1 CLASS ACTIVITY/HOME ACTIVITY A

1.1	$A = P(1+i)^n$ $= 800000(1.08)^5$ $= R1175462{,}46$ $\therefore R1175462{,}46 - R200\ 000$ $= R975462{,}46$ Some calculators give R 975 462,50	✓ substitution ✓ R 1 175 462,46 ✓ R 975 462,46 (3) Incorrect Formula: 0/3
1.2	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $800000 = \dfrac{10000\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\frac{0{,}14}{12}}$ $1-\left(1+\frac{0{,}14}{12}\right)^{-n} = \frac{14}{15} \quad (= 0{,}933333)$ $\left(1+\frac{0{,}14}{12}\right)^{-n} = \frac{1}{15} \quad (= 0{,}06666666)$ $\log\left(1+\frac{0{,}14}{12}\right)^{-n} = \log\frac{1}{15}$ $-n\log\left(1+\frac{0{,}14}{12}\right) = \log\frac{1}{15} \quad \left(-n = \dfrac{\log\frac{1}{15}}{\log\left(1+\frac{0{,}14}{12}\right)} = -233{,}47\right)$ $n = 233{,}47$ $\therefore$ the loan will be paid off at the end of the 234^{th} month	✓ substitute into P_v ✓ $i = \frac{0{,}14}{12}$ ✓ using logs ✓ answer (4)
	OR Balance outstanding after 233^{rd} month $= 800000\left(1+\frac{0{,}14}{12}\right)^{233} - \dfrac{10000\left[\left(1+\frac{0{,}14}{12}\right)^{233}-1\right]}{\frac{0{,}14}{12}}$ = R4 660,04 which is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitution into P formula ✓ 234 ✓ answer ✓ argument (4)
	OR Total value of the loan after 234 payments $= \dfrac{10000\left(1-\left(1+\frac{0{,}14}{12}\right)^{-234}\right)}{\frac{0{,}14}{12}}$ = R800 350,21 > R800 000 and the differences is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitution into F formula ✓ 234 ✓ answer ✓ argument (4)

1.3	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log\frac{1}{2} = n\log 0{,}93$ $n = \frac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years **OR** $A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log_{0{,}93}\frac{1}{2} = n$ $n = 9{,}55$ years **Note:** If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks **Note:** If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$	✓ $A = \frac{P}{2}$ ✓ subs into correct formula ✓ log ✓ answer (4)

13.2 CLASS ACTIVITY/HOME ACTIVITY B

1.1	Selling price / *Verkoopprys* $= \frac{102\ 000}{0{,}12}$ $= 850\ 000$	✓ 850 000 (1)
1.2	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $748\ 000 = \frac{x\left[1-\left(1+\frac{0{,}09}{12}\right)^{-240}\right]}{\frac{0{,}09}{12}}$ $x = 6\ 729{,}95$ **OR** $F_v = \frac{x[(1+i)^n - 1]}{i}$ $748\ 000\left(1+\frac{0{,}09}{12}\right)^{240} = \frac{x\left[\left(1+\frac{0{,}09}{12}\right)^{240}-1\right]}{\frac{0{,}09}{12}}$ $x = 6\ 729{,}95$	✓ $Pv = 748\ 000$ ✓ $i = \frac{0{,}09}{12}$ ✓ $n = -240$ ✓ $x = R6\ 729{,}95$ (4) ✓ $748000\,(1+\frac{0{,}09}{12})^{240}$ ✓ $i = \frac{0{,}09}{12}$ ✓ $n = 240$ ✓ $x = R6\ 729{,}95$ (4)
1.3	$\text{Balance} = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{6729{,}95\left[1-\left(1+\frac{0{,}09}{12}\right)^{-155}\right]}{\frac{0{,}09}{12}}$ $x = 615\ 509{,}74$	✓ 6729,95 ✓ $n = -155$ ✓ R615 509,74 (3)

13.3 Class Activity/Home Activity C

3.	$A = P(1-i)^n$ $72\ 500 = 145\ 000\,(1-i)^5$ $i = 1 - \sqrt[5]{\dfrac{72500}{145000}}$ $= 0{,}1294\ldots$ $\therefore$ Rate of interest/*Rentekoers* is 12,94 % p.a./*p.j.*	✓substitution/*substitusie* ✓writing in terms of i *herskryf in terme van i* ✓answer/*antwoord* (3)
3.2. 7.2.2	$P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $500000 = \dfrac{6000\left[1-\left(1+\dfrac{0{,}12}{12}\right)^{-n}\right]}{\dfrac{0{,}12}{12}}$ $\dfrac{500000}{6000} \times 0{,}01 = 1-(1{,}01)^{-n}$ $(1{,}01)^{-n} = 1 - \dfrac{5}{6}$ $-n = \dfrac{\log\dfrac{1}{6}}{\log 1{,}01}$ $n = 180{,}07$ Mphekgoane settles the loan in 181 months	✓6000 ✓substitute into correct formula/*substitusie in korrekte formule* ✓ use of logs/*gebruik van logs* ✓answer/*antwoord* (4)
3.2.	Malodi ~~Samuel~~ He is paying off his loan over a longer period thus more interest will be paid./*Hy betaal sy lening oor 'n langer tydperk af, dus sal hy meer rente betaal.* **OR/*OF*** Malodi ~~Samuel~~ He will pay/*Hy betaal* R5505,43 × 240 – R500 000 = R821 303,20 She will pay between/*Sy sal tussen* R580 000 and/*en* R586 000,00 *betaal*.	Malodi ~~Samuel~~ ✓reason/*rede* (2) Malodi ~~Samuel~~ ✓reason/*rede* (2) **[13]**

13.4 CLASS ACTIVITY/HOME ACTIVITY D

4.1.1	$400\times(44\times12)$ $=R211200$	✓ $R400\times(44\times12)$ ✓ $R211200$ (2)
4.2.	$F=\dfrac{x\left[(1+i)^n-1\right]}{i}$ $=\dfrac{400\left[\left(1+\frac{0,08}{12}\right)^{528}-1\right]}{\frac{0,08}{12}}$ $=R1\,943\,524,42$	✓ $x=400$ ✓ $n=528$ ✓ $i=\dfrac{0,08}{12}$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (5)
4.3	$P=\dfrac{x[1-(1+i)^{-n}]}{i}$ $2000000=\dfrac{x\left[1-\left(1+\frac{0,1}{12}\right)^{-300}\right]}{\frac{0,1}{12}}$ $x=R18\,174,01$ **OR/*OF*** $2000000\left(1+\dfrac{0,1}{12}\right)^{300}=\dfrac{x\left[\left(1+\frac{0,1}{12}\right)^{300}-1\right]}{\frac{0,1}{12}}$ $x=R18174,01$	✓ P = 2000000 ✓ $n=300$ and/*en* $i=\dfrac{0,1}{12}$ ✓ substituting into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (4) ✓ P = 2000000 ✓ $n=300$ and/*en* $i=\dfrac{0,1}{12}$ ✓ equating/stel gelyk ✓ answer/*antwoord* (4)

13.5 Class Activity/Home Activity E

1.1	$A = P(1+i)^n$ $12\ 146{,}72 = 10\ 000\left(1+\frac{r}{12}\right)^{36}$ $\left(1+\frac{r}{12}\right)^{36} = 1{,}214672$ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ $= 1{,}005416$ $\frac{r}{12} = 0{,}005416$ $r = 0{,}06500$ $r = 6{,}5\%$	✓ $\frac{r}{12}$ ✓ $n = 36$ ✓ correct substitution into formula ✓ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ ✓ 6,5% (5)
1.2.	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $235\ 000 = \frac{x\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}{\frac{0{,}11}{12}}$ $x = \frac{235\ 000 \times \frac{0{,}11}{12}}{\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}$ $= R5\ 536{,}95$ His monthly instalment is R 5 536,95	✓ $i = \frac{0{,}11}{12}$ ✓ $n = 54$ ✓ correct substitution in P ✓ answer (4)
1.2.2	Amount paid for the year : $(5\ 536{,}95 \times 12) = R66\ 443{,}40$ $\text{Balance} = 235\ 000\left(1+\frac{0{,}11}{12}\right)^{12} - \frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ $= 192\ 296{,}17$ $\text{Interest} = (5\ 536{,}95 \times 12) - (235\ 000 - 192\ 296{,}17)$ $= 66\ 443{,}40 - 42\ 703{,}83$ $= 23\ 739{,}57$	✓ R66 443,40 ✓ $235\ 000\left(1+\frac{0{,}11}{12}\right)^{12}$ ✓ $\frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 (6)

14. COGNITIVE LEVELS 3&4 QUESTIONS

14.1 PRESENT VALUE ANNUITY

14.1.1 Class Activity/Home Activity A

Malodi bought a house for R980 000.he paid a deposit of 10% of the selling price of the house. He obtained a loan from the bank at an interest rate of 11% per annum, compounded monthly, to pay the balance of the selling price. He agreed to pay monthly instalments of R10 000 on the loan.

1.1 How much money did Malodi borrow from the bank? (2)

1.2 How many months will it take to repay the loan? (6)

1.3 Calculate the balance of his loan immediately after his 90^{th}instalment. (3)

1.4 Malodi experienced financial difficulties after the 90^{th}instalment and did not pay the 91^{st} to the 95^{th}instalment. At the end of the 96^{th} month he increased his monthly instalment so as to pay off the loan in the same time interval as planned initially. Calculate the value of his new monthly instalment. (5)

14.1.2 Class Activity/Home Activity B

On 1 June 2016 a bank granted Thabiso a loan of R250 000 at an interest rate of 15% p.a.compounded monthly, to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2016 and ending 4 years later on 1 June 2020. However, Thabiso was unable to make the first two instalments and only commenced with the monthly instalments on 1 September 2016.

2.1 Calculate the amount Thabiso owed the bank on 1 august 2016, a month before his first monthly instalment (2)

2.2 Having paid the first monthly instalment on 1 September 2016, Thabiso will still pay his last monthly instalment on 1 June 2020.calculate his monthly instalment (4)

2.3 If Thabiso paid R9 000 as his monthly instalment starting on 1 September 2016, how many months sooner will he repay the loan? (5)

2.4 If Thabiso paid R9 000 as a monthly instalment starting on 1 September 2016, calculate the final instalment to repay the loan. (4)

14.1.3 Class Activity/Home Activity C

Galane takes a loan from a bank to buy a car for R235 000.he agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a. compounded monthly.

3.1.1 Calculate Galane's monthly instalment. (4)

3.1.2 Calculate the total amount of interest that Galane will pay during the first year of the repayment of the loan. (6)

14.1.4 Class Activity/Home Activity D

On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

4.1.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)

4.1.2 how many instalments of R3 200 MUST SHE PAY? (5)

4.1.3 calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan (5)

4.2. Nine years ago, a bank granted Matsobane a home loan of R525 000.this loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Matsobane's monthly repayments commenced exactly one month after the loan was granted.

4.2.1 Matsobane decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan (5)

14. 2 FUTURE VALUE

14.2.1 Class Activity/Home Activity A

1.1.Refilwe started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into annuity fund. She continued doing this until she retired on 31 December 2013.

1.1.1. Determine the total amount of money that she paid into the fund. (2)

1.1.2. The interest rate on this fund was 8% p.a., compound monthly. Calculate the value of the fund at the time that she retired. (5)

1.1.3. On the 1 January 2014 Refilwe invested R2 million in an account paying interest at 10% p.a. compounded monthly. Refilwe withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If Refilwe wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount she could withdraw at the end of each month. (4)

1.2. For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year. At the end of 2012 the populations of these two towns were equal. Determine the ratio of the population of town X (call it P_x) to the population of town Y (call it P_y) at the beginning of 2010. (4)

[15]

14.2.2 Class Activity/Home Activity B

2.1. On the January 2020, Mr Modiba made the first of his monthly deposits of R1 000 into a saving account. He continues to make monthly deposits of R1 000 at the end of each month until 31 January 2023. The interest rate was fixed at 7.5% p.a., compounded monthly.

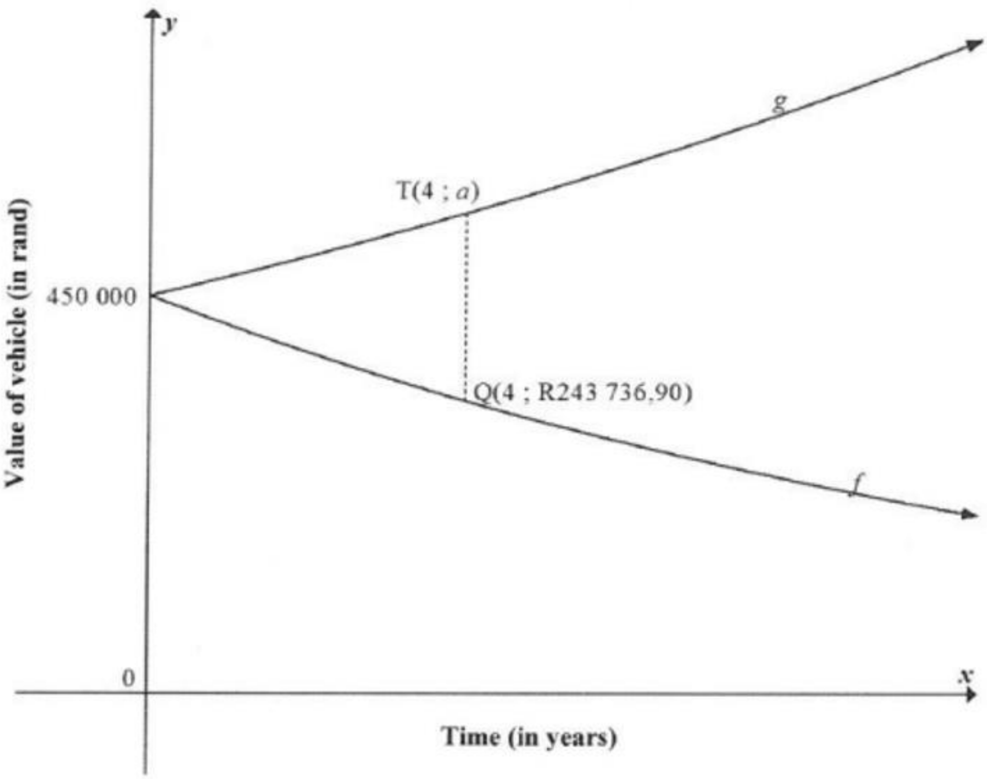
2.1.1. What will the investment be worth immediately after the last deposit? (4)

2.1.2. If he makes no further payments but leaves the money in the account, how Much money will be in the account on 31 January 2023? (2)

14.2.3 Class Activity/Home Activity C

The graph of f shows the book value of a vehicle x years after the time Malose bought it.

The graph of g shows the cost price of a similar new vehicle x year later.



3.1.How much did Malose pay for the vehicle? (1)

3.2. Use the reducing –balance method to calculate the percentage annual rate of depreciation of the vehicle that Malose bought. (4)

3.3.If the average rate of price increases of the vehicle is 8.1% p.a., calculate the value of a. (3)

3.4.A vehicle that cost R450 000 now, is to be replaced at the end of 4 years. The old Vehicle will be used as a trade-in. A sinking fund is created to cover the replacement Cost of this vehicle. Payment will be made at the end of each month. The first Payment will be made at the end of the 31^{th} month and the last payment will be made At the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.
Calculate the monthly payment to the fund. (5)

14.2.4 Class Activity/Home Activity D

4.1. On the 2^{nd} day of January 2015, a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance Method.
- When the book value of the printer is R49 152, the company will replace the Printer

4.1.1. Calculate the book value of the printer on the 2nd day of January 2017. (3)

4.1.2. At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)

4.1.3. The cost of a similar printer will be R280 000 at the beginning of 2020. the company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months after until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)

14.2.5 Class Activity/Home Activity E

5.1. Solly decided today that he will save R15 000 per quarter over the four years. He will make the first deposit into a savings account in three months' time he will make his last deposit at the end of four years from now.

5.1.1. How much will Solly have at the end of four years if interest is earned at 8.8% per annum, Compounded quarterly? (3)

5.1.2. If Solly decides to withdraw R100 000 from the account at the end of three years from now, how much will he have in the account at the end of four years from now? (3)

14.3 SIMPLE AND COMPOUND INTEREST

14.3.1 Class Activity/Home Activity A

Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8.3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.

1.1 Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations. (5)

14.3.2 Class Activity/Home Activity B

2.1 How many years will it take for an article to depreciate to half its value according to the Reducing-balance method at 7% per annum? (4)

2.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. they invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of 5 years, Radesh Will receive a bonus of exactly of the principal amount.
- Thandi: 8% compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer With appropriate calculations. (6)

2.3 Nicky opened a savings account with a single deposit of R7 00 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012.the account earns interest at 15% per annum compounded Monthly.

Determine the amount that should be in her savings account immediately after her last Deposit is made (that is on the 30 September 2012) (6)

15.1 Class Activity/Home Activity A

1.1	$0{,}10 \times R980\ 000$ $= R98\ 000$ $\therefore \text{Loan} = 980\ 000 - 98\ 000$ $= R882\ 000$	✓ $0{,}10 \times R980\ 000$ ✓ $R882\ 000$ (2)
1.2	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $882\ 000 = \dfrac{10\ 000\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-n}\right]}{\dfrac{0{,}11}{12}}$ $\dfrac{1617}{2000} = 1-\left(1+\dfrac{0{,}11}{12}\right)^{-n}$ $\left(1+\dfrac{0{,}11}{12}\right)^{-n} = \dfrac{383}{2000}$ $-n = \log_{\left(1+\frac{0{,}11}{12}\right)} \dfrac{383}{2000}$ $-n = -181{,}14$ $n = 181{,}14$ $\therefore$ It takes 182 months **OR**	✓ $i = \dfrac{0{,}11}{12}$ ✓ substitution into the correct formula ✓ $\left(1+\dfrac{0{,}11}{12}\right)^{-n} = \dfrac{383}{2000}$ ✓ introducing logs $-n = \log_{\left(1+\frac{0{,}11}{12}\right)} \dfrac{383}{2\ 000}$ or $-n = \log_{\left(\frac{1211}{1200}\right)} 0{,}1915$ or $-n = \dfrac{\log 0{,}1915}{\log 1{,}00916667}$ ✓ $n = 181{,}14$ ✓ 182 months (6)
	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $882\ 000 = \dfrac{10\ 000\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-n}\right]}{\dfrac{0{,}11}{12}}$ $\dfrac{383}{2000} = \left(1+\dfrac{0{,}11}{12}\right)^{-n}$ $\log\left(\dfrac{383}{2000}\right) = \log\left(1+\dfrac{0{,}11}{12}\right)^{-n}$ $\log\left(\dfrac{383}{2000}\right) = -n\log\left(1+\dfrac{0{,}11}{12}\right)$ $-n = -181{,}14$ $n = 181{,}14$ $\therefore$ It takes 182 months	✓ $i = \dfrac{0{,}11}{12}$ ✓ substitution into the correct formula ✓ $\dfrac{383}{2000} = \left(1+\dfrac{0{,}11}{12}\right)^{-n}$ ✓ introducing logs $-n = \dfrac{\log\left(\dfrac{383}{2000}\right)}{\log\left(1+\dfrac{0{,}11}{12}\right)}$ or $-n = \dfrac{\log 0{,}1915}{\log 1{,}00916667}$ ✓ $n = 181{,}14$ ✓ 182 months (6)

1.3	$n = 181{,}1379918 - 90$ $= 91{,}1379918$ $P_v = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $P_v = \dfrac{10\ 000\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-91{,}1379918}\right]}{\dfrac{0{,}11}{12}}$ $= R615\ 991{,}70$	✓ n = 91,1379918 ✓ substitution into the correct formula ✓ answer (3)

	OR $A = P(1+i)^n$ $= 882\ 000\left(1+\dfrac{0{,}11}{12}\right)^{90}$ $A = R2\ 005\ 069{,}01$ $F_v = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $= \dfrac{10\ 000\left[\left(1+\dfrac{0{,}11}{12}\right)^{90} - 1\right]}{\dfrac{0{,}11}{12}}$ $F_v = R1\ 389\ 077{,}31$ Outstanding balance after 90 instalments: $= R2\ 005\ 069{,}01 - R1\ 389\ 077{,}31$ $= R615\ 991{,}70$	✓ substitution into the correct formula (or 2 005 069,01) ✓ substitution into the correct formula (or 1 389 077,31) ✓ answer (R615 991,70) (3)

1.4	$A = P(1+i)^n$ $= 615\ 991,70\left(1+\frac{0,11}{12}\right)^5$ $= 644\ 747,02$ $P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $644\ 747,02 = \frac{x\left[1-\left(1+\frac{0,11}{12}\right)^{-87}\right]}{\frac{0,11}{12}}$ $x = R10\ 786,84$	✓ substitution into the correct formula ✓ answer ✓ substitution into the correct formula ✓ n = 182-95 = 87 ✓ answer (5) **[16]**

15.2 Class Activity/Home Activity B

2.1	$A = P(1+i)^n$ $= 250000\left(1+\frac{0,15}{12}\right)^2$ $= R256\ 289,06$	✓substituting i and n values in correct formula ✓ answer (2)
2.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $256\ 289,06 = \frac{x\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $3203,6133 = x\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]$ $x = R\ 7\ 359,79$ per month **OR/*OF*** $250000 = \frac{x\left(1+\frac{0,15}{12}\right)^{-2}\left[1-\left(1+\frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $x = R\ 7\ 359,79$	✓ $i = \frac{0,15}{12}$ ✓ $n = 46$ ✓ substitution into correct formula ✓ answer (4) ✓ $i = \frac{0,15}{12}$ ✓ $n = 46$ ✓ substitution into correct formula ✓ answer (4)
2.	$256\ 289,06 = \frac{9\ 000\left[1-\left(1+\frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\left(1+\frac{0,15}{12}\right)^{-n} = 0,6440429722$ $-n\log\left(1+\frac{0,15}{12}\right) = \log 0,6440429722$ $n = 35,41872568$ months/ *maande* $\therefore$ 36 payments are required $\therefore$ 36 *paaiemente moet betaal word* $\therefore$ Thabiso will pay his loan off 10 months sooner./Thabiso *los sy lening 10 maande vroeër af.* **OR/*OF***	✓$x = 9\ 000$ ✓ substitute into correct formula ✓ use of logs ✓ $n = 35,42$ ✓ 10 months (5)

	$256289{,}06\left(1+\frac{0{,}15}{12}\right)^n=\frac{9000\left[\left(1+\frac{0{,}15}{12}\right)^n-1\right]}{\frac{0{,}15}{12}}$ $3203{,}61325\left(1+\frac{0{,}15}{12}\right)^n=9000\left(1+\frac{0{,}15}{12}\right)^n-9000$ $9000=5796{,}38675\left(1+\frac{0{,}15}{12}\right)^n$ $n=\log_{\left(1+\frac{0{,}15}{12}\right)}1{,}5523691425$ $n=35{,}41872568$ $\therefore$ 36 payments are required $\therefore$ *36 paaiemente moet betaal word* $\therefore$ Thabiso will pay his loan off 10 months sooner./Thabiso *los sy lening 10 maande vroeër af.*	✓ 9 000 ✓ substitute into correct formula ✓ use of logs ✓ $n=35{,}42$ ✓ 10 months (5)
2.	The balance of his loan after the 35th payment was made: *Die balans van sy lening nadat die 35ste paaiement betaal is:* $\text{Balance}=256289{,}06\left(1+\frac{0{,}15}{12}\right)^{35}-\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{35}-1\right)}{\frac{0{,}15}{12}}$ $=\text{R }3\ 735{,}45$ $\text{Final instalment}=3\ 735{,}45\left(1+\frac{0{,}15}{12}\right)$ $=\text{R }3\ 782{,}14$	✓ $256289{,}06\left(1+\frac{0{,}15}{12}\right)^{35}$ ✓ $\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{35}-1\right)}{\frac{0{,}15}{12}}$ ✓ $3\ 735{,}45\left(1+\frac{0{,}15}{12}\right)$ ✓ answer (4)
	OR/OF $P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ Final instalment $=\frac{9\ 000\left[1-\left(1+\frac{0{,}15}{12}\right)^{-0{,}41872568}\right]}{\frac{0{,}15}{12}}\left(1+\frac{0{,}15}{12}\right)$ $=\text{R }3\ 782{,}14$	✓ 0,41872568 ✓ $\frac{9\ 000\left[1-\left(1+\frac{0{,}15}{12}\right)^{-0{,}41872568}\right]}{\frac{0{,}15}{12}}$ ✓ $\times\left(1+\frac{0{,}15}{12}\right)$ ✓ answer (4)
	OR/OF	
	$\text{Balance}=256289{,}06\left(1+\frac{0{,}15}{12}\right)^{36}-\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{36}-1\right)}{\frac{0{,}15}{12}}$ $=\text{R }-5\ 217{,}86$ $\text{Final payment}=9\ 000-5217{,}86$ $=\text{R }3\ 782{,}14$	✓ $256289{,}06\left(1+\frac{0{,}15}{12}\right)^{36}$ ✓ $\frac{9000\left(\left(1+\frac{0{,}15}{12}\right)^{36}-1\right)}{\frac{0{,}15}{12}}$ ✓ $9\ 000-5217{,}86$ ✓ answer (4) **[15]**

15.3 Class Activity/Home Activity C

3.1.1	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $235\ 000=\frac{x\left[1-\left(1+\frac{0,11}{12}\right)^{-54}\right]}{\frac{0,11}{12}}$ $x=\frac{235\ 000\times\frac{0,11}{12}}{\left[1-\left(1+\frac{0,11}{12}\right)^{-54}\right]}$ $=R5\ 536,95$ His monthly instalment is R 5 536,95	✓ $i=\frac{0,11}{12}$ ✓ $n=54$ ✓ correct substitution in P ✓ answer (4)
3.1.2	Amount paid for the year : $(5\ 536,95\times12)$ = R66 443,40 $\text{Balance}=235\ 000\left(1+\frac{0,11}{12}\right)^{12}-\frac{5\ 536,95\left[\left(1+\frac{0,11}{12}\right)^{12}-1\right]}{\frac{0,11}{12}}$ $=192\ 296,17$ $\text{Interest}=(5\ 536,95\times12)-(235\ 000-192\ 296,17)$ $=66\ 443,40\ -\ 42\ 703,83$ $=23\ 739,57$ **OR/*OF***	✓ R66 443,40 ✓ $235\ 000\left(1+\frac{0,11}{12}\right)^{12}$ ✓ $\frac{5\ 536,95\left[\left(1+\frac{0,11}{12}\right)^{12}-1\right]}{\frac{0,11}{12}}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 **OR/*OF***

	Total amount paid in first year $= R\,5\,536.95 \times 12$ $= R66\,443,40$ Balance on loan after 1 year = P of remaining installments $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{5\,536,95\left[1-\left(1+\frac{0,11}{12}\right)^{-42}\right]}{\frac{0,11}{12}}$ $= R192\,296,20$ Amount paid off in the first year: $R235\,000 - R192\,296,20 = R42\,703,80$ Amount of interest $= R66\,443,40 - R42\,703,80$ $= R23\,739,60$ **OR/*OF*** $P = \frac{5536,95\left[1-\left(1+\frac{0,11}{12}\right)^{-12}\right]}{\frac{0,11}{12}}$ $= R\ 62\,648,18$ $235\,000 - 62\,648,18 = R172\,351,82$ After 12 months, money owed on house is $172\,351,82\left(1+\frac{0,11}{12}\right)^{12}$ $= 192\,296,17$ Amount paid after 12 months is $5\,536,95 \times 12 = R\ 66\,443,\ 40$ Amount of interest paid: $R\ 66\,443,\ 40 - (235\,000 - 192\,296,17)$ $= R\ 23\,739,\ 57$	✓ R66 443,40 ✓ $n = -42$ ✓ substitution into correct formula ✓ R192 296,20 ✓ R42 703,80 ✓ R23 739,60 (6) **OR/*OF*** ✓ R62 648,18 ✓ R172 351,82 ✓ R192 296,17 ✓ R66 443,40 ✓ 235 000 – 192 296,17 ✓ R23 739,57 (6) **[15]**

15.4 Class Activity/Home Activity D

4.1.1	After eleven months, Genevieve will owe/ *Na elf maande skuld Genevieve* $A = 82\ 000\left(1+\frac{0,15}{12}\right)^{11}$ $= R\ 94006,79$	✓ $n = 11$ ✓ correct substitution into correct formula ✓ answer (3)
4.1.	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $94006,79 = \frac{3\ 200\left[1-\left(1+\frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\frac{94\ 006,79}{3\ 200}\times\frac{0,15}{12} = 1-\left(1+\frac{0,15}{12}\right)^{-n}$ $\left(1+\frac{0,15}{12}\right)^{-n} = 1-0,3672147...$ $-n\log\left(1+\frac{0,15}{12}\right) = \log 0,6327852...$ $-n = -36,8382...$ $n = 36,84$ Genevieve will have to pay 36 installments of R3 200	✓ 94006,79 ✓ substitute into correct formula ✓correct use of logs (logs to be defined) ✓ n =36,84 ✓ 36 installments (5)

4.1.	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $=\frac{3200\left[1-\left(1+\frac{0,15}{12}\right)^{-0,83826912}\right]}{\frac{0,15}{12}}$ $P=2652$ Outstanding balance after 36 installments is R2 652 Final payment will be: $A=2652,00\left(1+\frac{0,15}{12}\right)^{1}$ $=\text{R }2685,00$ **OR/*OF*** $\text{Balance}: 94006,79\left(1+\frac{0,15}{12}\right)^{36}-\frac{3\,200\left[\left(1+\frac{0,15}{12}\right)^{36}-1\right]}{\frac{0,15}{12}}$ $=\text{R}2\ 651,72$ Final payment will be: $A=2651,72\left(1+\frac{0,15}{12}\right)^{1}$ $=\text{R }2\,685,00$	✓ $n=-083826912$ ✓substitute into correct formula ✓answer ✓ $2652,00\left(1+\frac{0,15}{12}\right)^{1}$ ✓answer **OR/*OF*** ✓ $94006,79\left(1+\frac{0,15}{12}\right)^{36}$ ✓ $\frac{3\,200\left[\left(1+\frac{0,15}{12}\right)^{36}-1\right]}{\frac{0,15}{12}}$ ✓ 2 651,72 ✓ $2651,72\left(1+\frac{0,15}{12}\right)^{1}$ ✓answer (5) **[16]**

4.2.1	$P=\frac{x[1-(1+i)^{-n}]}{i}$ $525\ 000=\frac{6\ 000\left[1-\left(1+\frac{0,1}{12}\right)^{-n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48}=1-\left(1+\frac{0,1}{12}\right)^{-n}$ $-n\log\left(1+\frac{0,1}{12}\right)=\log\frac{13}{48}$ $-n=\frac{\log\frac{13}{48}}{\log\left(1+\frac{0,1}{12}\right)}$ $n=157{,}40$ n = 158 payments **OR/OF**	✓ $\frac{0,1}{12}$ ✓substitution into the correct formula ✓simplification ✓use of logs ✓answer (5) **OR/OF**

	$P=\frac{x[1-(1+i)^{-n}]}{i}$ $525\ 000=\frac{6\ 000\left[1-\left(1+\frac{0,1}{12}\right)^{-12n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48}=1-\left(1+\frac{0,1}{12}\right)^{-12n}$ $-12n\log\left(1+\frac{0,1}{12}\right)=\log\frac{13}{48}$ $-12n=\frac{\log\frac{13}{48}}{\log\left(1+\frac{0,1}{12}\right)}$ $n=\frac{\log\frac{13}{48}}{\log\left(1+\frac{0,1}{12}\right)}\times\frac{1}{12}$ $n=13{,}11686841$ Number of payments $=13{,}11686841\times 12=157{,}40$ n = 158 payments	✓ $\frac{0,1}{12}$ ✓substitution into the correct formula ✓simplification ✓use of logs ✓answer (5)

4.2.2	Difference: R6 000 – R5 066,36 = R933,64 $F=\frac{x[(1+i)^n-1]}{i}$ $F=\frac{933{,}64\left[\left(1+\frac{0{,}1}{12}\right)^{108}-1\right]}{\frac{0{,}1}{12}}$ $=R162\,503{,}51$	✓ R933,64 ✓ $n=108$ ✓ substitution into the correct formula ✓ answer (4)

16.1 Class Activity/Home Activity A

1.1.	$R400\times(44\times12)$ $=R211200$	✓ $R400\times(44\times12)$ ✓ $R211200$ (2)
1.1.	$F=\dfrac{x\left[(1+i)^n-1\right]}{i}$ $=\dfrac{400\left[\left(1+\dfrac{0,08}{12}\right)^{528}-1\right]}{\dfrac{0,08}{12}}$ $=R1\ 943\ 524,42$	✓ $x=400$ ✓ $n=528$ ✓ $i=\dfrac{0,08}{12}$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (5)
1.1.	$P=\dfrac{x[1-(1+i)^{-n}]}{i}$ $2000000=\dfrac{x\left[1-\left(1+\dfrac{0,1}{12}\right)^{-300}\right]}{\dfrac{0,1}{12}}$ $x=R18\ 174,01$ **OR/*OF*** $2000000\left(1+\dfrac{0,1}{12}\right)^{300}=\dfrac{x\left(\left(1+\dfrac{0,1}{12}\right)^{300}-1\right)}{\dfrac{0,1}{12}}$ $x=R18174,01$	✓ $P=2000000$ ✓ $n=300$ and/*en* $i=\dfrac{0,1}{12}$ ✓ substituting into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (4) ✓ $P=2000000$ ✓ $n=300$ and/*en* $i=\dfrac{0,1}{12}$ ✓ equating/stel gelyk ✓ answer/*antwoord* (4)

1.	Let P_X and P_Y be the populations of the two towns at the beginning of 2010./*Laat* P_X *en* P_Y *die bevolkings wees van die twee dorpe aan die begin van 2010.* $A_X=A_Y$ $P_X(1-0,08)^3=P_Y(1+0,12)^3$ $\dfrac{P_X}{P_Y}=\dfrac{(1+0,12)^3}{(1-0,08)^3}$ $=\dfrac{1,404...}{0,778...}$ $=1,8:1$	✓ equating/*stel gelyk* ✓ $A_X=P_X(1-0,08)^3$ ✓ $A_Y=P_Y(1+0,12)^3$ ✓ answer/*antwoord* (4) **[15]**

16.2 Class Activity/Home Activity B

2.1.1	$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $=\frac{1\,000\left[\left(1+\frac{0{,}075}{12}\right)^{145}-1\right]}{\frac{0{,}075}{12}}$ $=R234\ 888{,}53$	✓ $n=145$ ✓ $i=\frac{0{,}075}{12}$ ✓ substitution into the correct formula ✓ answer (4)
2.1.2	$A=P(1+i)^n$ $=234\ 888{,}53\left(1+\frac{0{,}075}{12}\right)^{12}$ $=R253\ 123{,}54$	✓ substitution into the correct formula ✓ answer (2)

16.3 Class Activity/Home Activity C

3.1	R450 000	✓answer (1)
3.2	$A=P(1-i)^n$ $f(x)=450000(1-i)^x$ $243\ 736{,}90=450000(1-i)^4$ $i=1-\sqrt[4]{\frac{243\ 736{,}90}{450000}}$ $i=0{,}1421$ The rate of depreciation is 14,21% p.a. *Die waardeverminderingskoers is 14,21% p.j.*	✓substitution of 450 000 into correct formula ✓substitution of $(4;\ 243\ 736{,}90)$ into correct formula ✓making i the subject ✓answer (4)
3.3	At T : $A=P(1+i)^n$ $g(x)=450000(1+i)^x$ $a=450000(1+0{,}081)^4$ $=R614490{,}66$	✓$i=0{,}081$ & $n=4$ ✓correct substitution into formula ✓answer (3)

3.4	$\text{Future Value} = R614\,490,66 - R243\,736,90$ $= R370\,753,76$ Let x be the value of monthly payment $F_v = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $370753,76 = \dfrac{x\left[\left(1+\frac{0.062}{12}\right)^{36} - 1\right]}{\frac{0.062}{12}}$ $x = R9397,11$	✓R370 753,76 ✓$i = \dfrac{0,062}{12}$ ✓$n = 36$ ✓substitution into correct formula ✓answer (5) **[13]**

16.4 Class Activity/Home Activity D

4.1.1	$A = 150\,000(1-0,2)^2$ $= R96\,000$	✓ $n = 2$ ✓ 150 000 in correct formula ✓ 96 000 (3)
4.1.	$150\,000(1-0,2)^n = 49\,152$ $(0,8)^n = \dfrac{1024}{3125}$ $n\log(0,8) = \log\dfrac{1024}{3125}$ $n = 5$ The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word* **OR / OF** $150\,000(1-0,2)^n = 49\,152$ $(0,8)^n = \dfrac{1024}{3125}$ $n = \log_{0,8}\dfrac{1024}{3125}$ $n = 5$ The machine will need to be replaced at the beginning of 2020 / *Masjien moet aan die begin van 2020 vervang word*	✓ $150\,000(1-0,2)^n = 49\,152$ ✓ $n\log(0,8) = \log\dfrac{1024}{3125}$ ✓ $n = 5$ ✓ 2020 (4) ✓ $150\,000(1-0,2)^n = 49\,152$ ✓ $n = \log_{0,8}\dfrac{1024}{3125}$ ✓ $n = 5$ ✓ 2020 (4)
4.1.3	R280 000 – R49 152 =R230 848 $230\,848 = \dfrac{x\left[\left(1+\frac{0,085}{4}\right)^{20} - 1\right]}{\frac{0,085}{4}}$ $x = R9\,383,26$	✓ R230 848 ✓ $i = \dfrac{0,085}{4} = 0,02125$ and $n = 20$ ✓ subs into correct formula ✓ R 9 383,26 (4)

16.5 Class Activity/Home Activity E

5.1.	$F=\frac{x[(1+i)^n-1]}{i}$ $F=\frac{15\ 000\left[\left(1+\frac{0,088}{4}\right)^{16}-1\right]}{\frac{0,088}{4}}$ $F\ \ =\text{R}283\ 972,28$	✓ $\frac{0,088}{4}$ and $n=16$ ✓substitution into correct formula ✓answer (3)

5.1.2	$\text{A}=\text{R}283\ 972,28-100\,000\left(1+\frac{0,088}{4}\right)^4$ $=\text{R }174\ 877,60$	✓ future value – amount including interest ✓ $100\,000\left(1+\frac{0,088}{4}\right)^4$ ✓answer (3)
	OR/OF Amount at end of 3 years: $F=\frac{15\ 000\left[\left(1+\frac{0,088}{4}\right)^{12}-1\right]}{\frac{0,088}{4}}-100\,000$ $=\text{R}103\ 459,12$	**OR/OF** ✓ R15 000 including interest – R100 000
	Amount at end of 4 years: $P(1+i)^n+\frac{x[(1+i)^n-1]}{i}$ $=103\,459,12\left(1+\frac{0,088}{4}\right)^4+\frac{15000\left[\left(1+\frac{0,088}{4}\right)^4-1\right]}{\frac{0,088}{4}}$ $=\text{R }174\ 877,60$	✓ $\left(1+\frac{0,088}{4}\right)^4$ on P and x in F_v ✓method (3)

17. SIMPLE AND COMPOUND INTEREST SOLUTIONS

17.1 Class Activity/Home Activity A

6.1 1.1	Kuda : $A = P(1+in)$ $= 5\ 000(1+0{,}083\times 4)$ $= R6\ 660{,}00$ Final Answer : R6 660,00 + R266,40 $= R6\ 926{,}40$	✓substitution into the correct formula ✓final answer
	OR/OF Kuda : $A = P(1+in)\times 1{,}04$ $= 5\ 000(1+0{,}083\times 4)\times 1{,}04$ $= R6\ 926{,}40$	**OR/OF** ✓substitution into the correct formula ✓final answer
	Thabo : $A = P(1+i)^n$ $= 5\ 000\left(1+\frac{0{,}081}{12}\right)^{12\times 4}$ $= R6\ 905{,}71$	✓substitution into the correct formula ✓answer
	Kuda will have a better investment	✓conclusion (5)

17.2 Class Activity/Home Activity B

2.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log\frac{1}{2} = n\log 0{,}93$ $n = \frac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years **Note:** If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks	**OR**	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log_{0{,}93}\frac{1}{2} = n$ $n = 9{,}55$ years **Note:** If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$	✓ $A = \frac{P}{2}$ ✓ subs into correct formula ✓ log ✓ answer (4)

2.2	**Radesh:** $A = P(1+in)$ $= 6\,000(1+0{,}085\times 5)$ $= 8\,550$ OR $A = 6\,000 + 8{,}5\%\text{ of }6000\times 5$ $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\,550$ $\text{Bonus} = 0{,}05\times 6\,000$ $= 300$ $\text{Received} = 8\,550 + 300$ $= \text{R}8\,850$ **Thandi:** $A = P(1+i)^n$ $= 6\,000\left(1+\frac{0{,}08}{4}\right)^{20}$ $= \text{R}8\,915{,}68$ Thandi's investment is bigger.	✓ 8 550 ✓ **R**8 850 ✓ $n = 20$ ✓ $i = \frac{0{,}08}{4}$ ✓ answer ✓ choice made (6)
2.3	F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$ **OR** F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 11220{,}68\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$	✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ ✓ answer (6) ✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ ✓ answer (6)

	OR $$F_v = 300\left(1+\frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$$ $= 375,17 + 14\ 907,74$ $= \text{R}15\ 282,91$	✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or $0,0125$ ✓ $n = 19$ (corresponding to 700) ✓ $n = 18$ (corresponding to 300) ✓ $300\left(1+\frac{0,15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$ ✓ answer (6) **[16]**

Capricorn North District

Mathematics

Calculus

Grade 12

Manual Activities

Level 1 & 2

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1. LIMITS

The Limit of a function is denoted as : $\lim_{x \to a} f(x)$

The limit of a function is the value that the function tends to (approaches) as a variable in the function (for example x) tends to a specific value (for example a)

1.1. Examples of the questions on the limits

- Determine the value of $\lim_{x \to 2}(4x + 7)$

$\lim_{x \to 2}(4x + 7)$ $\rightarrow$ *it reads as: the limit of* $4x + 7$ *as x approaches* $2 == (4(2) + 7)$ $\rightarrow$ substitute the value that x approaches

=15

1.2. Differential Calculus

It is a mathematical method of getting the gradient of a given function or the rate of change. This method is called the derivative.

Differentiation

Now learners have learned the concept of average gradient. Then they will be introduced to the gradient at a point. **Using differentiation, it is possible to determine the gradient at a specific point.**

The two methods that can be used to determine the derivative of a function:

- Determine the derivative from first principles.
- Determine the derivative using the rules for differentiation.

FORMULA FOR THE GRADIENT AT A POINT (derivative from first principles):

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$f'(x)$is the notation used for the derivative, which represents the gradient at a point.

Do not confuse the derivative with f(x), which is the y-value of a function at specific x-value, or with $f^{-1}(x)$ which is the inverse of $f(x)$.

Functions can be given as equations OR expressions

For Expressions

- D_x , $\frac{\mathrm{d}}{\mathrm{d}x}$

For equations

- $\frac{dy}{dx}, f'(x)$

2.DETERMINING THE DERIVATIVE FROM THE FIRST PRINCIPLE:

Use limits to determine the derivative of a function f at any x:

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Common errors and misconceptions made by the learners in attempting to answer questions relating to the first principle.

- Some learners can make the following notational errors: $\lim\limits_{h\to 0} = \frac{f(x+h)-f(x)}{h}$ or left out the limit of h altogether. They also loose marks for these errors. Other learners make errors in substitution.

Suggestions for improvement: Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.

1.3. Questions for First Principle [LEVEL 2 QUESTIONS]

1. Determine $f'(x)$ from first principles if $f(x) = x^2 + x$ (5)
2. Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 1$ (5)
3. Determine $f'(x)$ from first principles if $f(x) = 4 - 7x$ (4)
4. Determine $f'(x)$ from first principles if $f(x) = -\frac{2}{x}$ (5)
5. Determine $f'(x)$ from first principles if $f(x) = x^3$ (5)

3. RULES FOR DIFFERENTIATION

The rules for differentiation allow you to differentiate functions without going through the process of differentiating from first principles. Unless you are asked to differentiate from first principles, use the rules for differentiation.

Notation: $f'(x)$; $D_x\ (**)$; $\frac{d}{dx}(**)$; $\frac{dy}{dx}$

Rule for derivative: If $f(x) = ax^n\ \ then\ f'(x) = (n \times a)x^{n-1}$

Use the formula, $\frac{dy}{dx}(ax^n) = (n \times a)x^{n-1}$

Rule 1- The derivative of a constant is equal to 0.

If $f(x) = k,$ where k is a constant real number, when $f'(x) = 0$.

Rule 2-The power rule.

If $f(x) = x^n$, where n is a constant real number, then $f'(x) = nx^{n-1}$

Rule 3- The derivative of a function multiplied by a constant.

If $f(x) = k.g(x)$, where k is a constant real number, then $f'(x) = k.g'(x)$.

- $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)],\ \ (k\ a\ constant$

Rule 4- The sum rule.

If $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$.

- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Rule 5- The difference rule.

If $f(x) = g(x) - h(x)$, then $f'(x) = g'(x) - h'(x)$.

1.4 Before differentiating remove the following:

- **Brackets**
- **Fractions** $(no\ variable\ (e.g\ x) must\ be\ in\ denominator: \frac{1}{x^n} = x^{-n})$
- **Surds**$(\sqrt{x}\ remore\ all\ radical\ signs, to\ exponential\ form: \sqrt[n]{x} = x^{\frac{1}{n}})$

Common errors and misconceptions for the rules of derivatives

Some learners incorrectly changed $7\sqrt[3]{x^2}\ to\ 7x^{\frac{3}{2}}$ instead of $7x^{\frac{2}{3}}$

Suggestions for improvement: the teachers should revise the rules of exponents and surds expression into differentiable format.

1.4 Questions of Rules for derivatives. [LEVEL 2 QUESTIONS]

1. Determine $f'(x)\ if\ f(x) = 2x^5 - 3x^4 + 8x$ (3)
2. Determine $f'(x)\ if\ f(x) = \frac{x+2x^3}{\sqrt{x}}$ (3)
3. Determine:

   a. $\frac{dy}{dx}(\sqrt[5]{x^2} + x^3)$ (3)

   b. $f'(x)\ if\ f(x) = \frac{4x^2-9}{4x+6}; x \neq -\frac{3}{2}$ (4)

2 Determine $\frac{dy}{dx}\ if$:

a. $y = 3x^3 + 6x^2 + x - 4$ (3)
b. $yx - y = 2x^2 - 2x$; $x \neq 1$ (4)
c. $y = 4x^8 + \sqrt{x^3}$ (3)
d. $y = (x^2 - \frac{1}{x^2})^2$ (3)
e. $y = \frac{1}{4}x^2 - 2x$ (2)
f. $y = 2x^{-4} - \frac{x}{5}$ (2)
g. $xy = 5$ (3)
h. $y = \frac{1-2x+x^2}{x^2}$ (3)

3 Given: $y = ax^2 + a$

Determine $\frac{dy}{dx}$ (1)

Determine $\frac{dy}{da}$ (2)

4 Determine $D_x(\frac{x^3-1}{x-1})$ (3)

5 Differentiate:

a. $f(x) = -3x^2 + 5\sqrt{x}$ (3)

b. $p(x) = (\frac{1}{x^3} + 4x)^2$

(4)

6 Given: $y = 4(\sqrt[3]{x^2})$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$ (4)

2.TANGENTS TO GRAPHS OF FUNCTIONS

A tangent is a straight line that touches a graph at one point. The gradient of the tangent is equal to the gradient of the graph at the point of contact. Thus, the derivative of the graph at the point of contact is equal to the gradient of the tangent.

2.1 Finding the equation to a tangent line

The slope of the tangent line to the graph at a point is equal to the derivative of the function at that point. So to find the equation of the tangent line to $f(x)\ at\ x = a$

- Take the derivative, and then
- Evaluate the derivative at $x = a\ i.e\ calculate\ f^{/}(a) = m$ to get the gradient of the tangent line at $x = a$
- Calculate the y-value at $x = a\ i.e\ calculate\ f(a)$, the corresponding y-value{point of contact i.e. ((a;f(a))}
- Lastly use the equation of the line: $y = mx + c\ or\ y - y_1 = m(x - x_1)$

2.2 Questions of the Tangents [LEVEL 3 QUESTIONS]

Determine the equation of the tangent to the curve at the given point

1. $y = x^3\ at\ the\ point\ x = -\frac{1}{2}$ (3)
2. $f(x) = x^3 - 6x^2 + 9x\ at\ the\ point\ x = 4$ (3)
3. $f(x) = x^3 - 2x^2 - x + 2\ at\ the\ point\ x = 1$ (3)
4. The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1; -7)$. For which values of x will g be concave up? (4)
5. The curve with equation $y = x + \frac{12}{x}$ passes through the point $A(2; b)$.

   Determine the equation of the line perpendicular to the tangent to the curve

   at A. (4)
6. Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$.
   Determine the interval on which f is concave up. (4)
7. $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$.
   Calculate the values of $a\ and\ b$. (5

Capricorn North District

Mathematics

Calculus

Grade 12

Manual Activities

Level 3 & 4

COGNITIVE LEVEL 3 & 4

6 SKETCHING CUBIC GRAPHS

6.1 FACTORISATION OF CUBIC POLYNOMIALS

To determine the x-intercepts of the cubic function we need to know how to factorise the cubic polynomials.

6.2 Three methods of factorising cubic polynomials will be shown in this activity manual

- Using inspection method
- Using synthetic division method
- Long division method

6.3 Steps to factorising cubic polynomials using inspection:

- Determine a factor of the polynomial using the factor theorem.
- Determine the coefficients of x^2 and the constant by inspection.
- Determine coefficient of x by equating coefficients.
- Factorise the quadratic factor in order to obtain the other factors of the cubic polynomial (the quadratic formula may have to be used).

6.4 Steps to factorising cubic polynomials using synthetic division:

- Determine a factor of the polynomial using the factor theorem.
- Use synthetic division to obtain a quadratic factor of the cubic polynomial.
- Factorise the quadratic in order to obtain the other factors of the cubic polynomial (the quadratic formula may have to be used).

7 CUBIC GRAPHS:

- Have one y-intercept
- Have at least one x-intercept, but at most three x-intercepts.
- May have stationary points which are local maximum and/ or local minimum point(s)
- May have a point of inflection which is a stationary point, or which is a point of inflection only.

The General equation: $y = ax^3 + bx^2 + cx + d$

- $y - intercept$: $Let\ x = 0$
- $x - intercepts/\ Roots$: $Let\ y = f(x) = 0$
    - Factorise through grouping or use the factor theorem, or any other method that can help solve cubic polynomial
    - If 2 roots are the same then, that double root is the Turning point
- Turning points
    - Determine $f'(x)$
    - Let $f'(x) = 0$
    - Solve for x
    - Substitute into original equation to find corresponding $y - value$.
    - Write as ordered pairs $(x_1\ ;f(x_1))$ & $(x_2\ ;f(x_2))$
- Sketch the graph following the three steps from above.

8 INCREASING AND DECREASING FUNCTIONS AND STATIONARY POINTS.

- The derivative is the gradient at a point of a function.

- If the gradient of a function is equal to zero, the tangent will be a straight horizontal line. At this point the graph will have a stationary point also known as the turning point
- At a turning point, the slope of the graph will change. The graph will either be increasing or decreasing as you approach the turning point. You can see whether the graph will be increasing or decreasing from the sign of the gradient.
- If the derivative is negative, the gradient will be negative and hence the graph will be decreasing. If the derivative is positive, the gradient will be positive and hence the graph will be increasing.

8.1 CONCAVITY

A function is said to be concave up on the interval $(a; b)$ if the function lies above tangents drawn to the curve at any point between a and b.

A function is said to be concave down on the interval $(a; b)$ if the function lies below tangents drawn to the curve at any point between a and b.

8.2 POINT OF INFLECTION

A point of inflection is a point on a curve where the concavity changes, either from concave up to concave down or vice versa.

The second derivative can be used to determine the concavity of a function. The

- Inflection points
- Determine $f''(x)$
- Let $f''(x)$=0
- Solve for x
- Substitute into original equation to find corresponding $y - value$.

9 EXAMINATION GUIDELINE

1. In respect of cubic functions, learners are expected to be able to:
   - Determine the equation of a cubic function from a given graph.
   - Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
   - Apply knowledge of transformations on a given function to obtain its image.
2. Learners are expected to be able to draw and interpret the graph of the derivative of a function.
3. Surface area and volume will be examined in the context of optimisation.
4. Learners must know the formulae for the surface area and volume of the right prisms. These formulae will NOT be provided on the formula sheet.
5. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Learners will be expected to select the correct formula from this list

10 COMMON ERRORS AND MISCONCEPTIONS

(a) In determining parameters (a, b, c and d) in $= ax^3 + bx^2 + cx + d$, learners must create linear equations and solve them simultaneously. Most learners manage to substitute the coordinates of the turning point/s into the given equation and obtain the first equation. They are unable to derive the second equation because it requires them to make use of the derivative. Some learners take a and b as given and use these values in the given equation to calculate the turning points of the function. This is considered a circular argument and is not acceptable.

(b) Some learners experience difficulty in factorising the equation in order to calculate the coordinates.

(c) Many learners fail to translate the words into mathematical language. They are unable to link an increasing function to where the value of y increases when moving from left to right on the x-axis. Learners have little idea that the change in concavity occurs at the point of inflection on the graph. Many do not calculate the x-coordinate of the point of inflection.

11 SUGGESTIONS FOR IMPROVEMENT

(a) The focus when teaching cubic functions should not only be on calculating the critical points but also on interpreting the critical points on the graph. For example, what does it mean when we know that the x-coordinate of a turning point on a graph is 4?

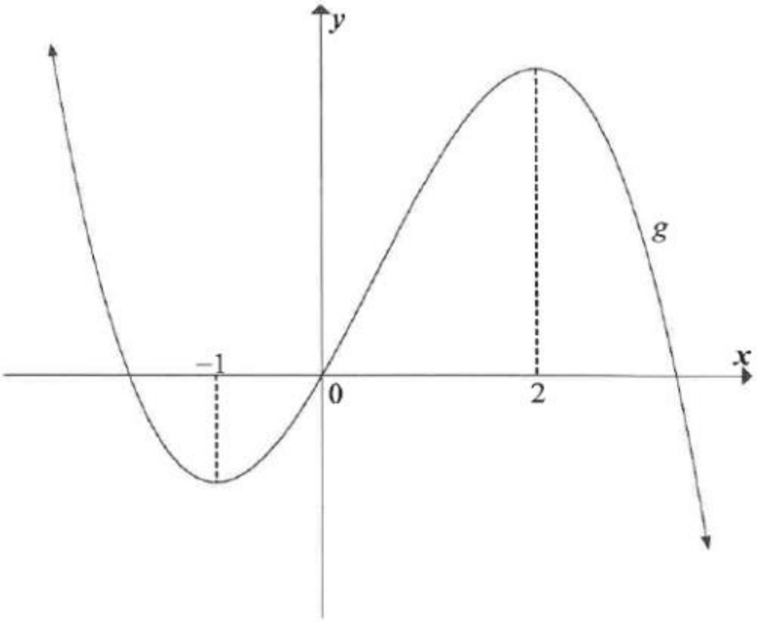
(b) When teaching graphs of cubic functions, teachers should inform learners of both methods of determining the x-coordinate of the point of inflection: solving for x in $F''(X)= 0$ as well as determining the x- value midway between the two turning points.

(c) Teachers should teach concavity in such a way that learners can visually identify where a graph is concave up or concave down. In this way, learners should deduce that the point of inflection is critical to establishing the concavity of a cubic graph.

12 COGNITIVE LEVEL 3 & 4 ACTIVITIES

12.1 ACTIVITY 1

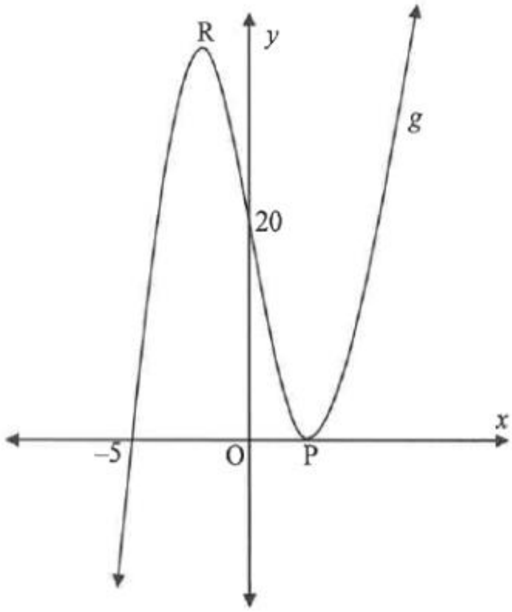
The graph of $g(x) = ax^3 + bx^2 + cx$, a cubic function having a y-intercept of 0, is drawn below. The x-coordinates of the turning points of g are -1 and 2.



1.1	For which values of x will g increase?	(2)
1.2	Write down the x-coordinate of the point of inflection of g.	(2)
1.3	For which values of x will g be concave down?	(2)
1.4	If $g'(x) = -6x^2 + 6x + 12$, determine the equation of g.	(4)
1.5	Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y = mx + c$.	(5)
		[15]

12.2 ACTIVITY 2

2.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5 ; 0)$ and at P, and the y-axis at $(0 ; 20)$.
P and R are turning points of g.



2.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

2.1.2 Calculate the coordinates of P and R. (5)

2.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)

2.2 If g is a cubic function with:

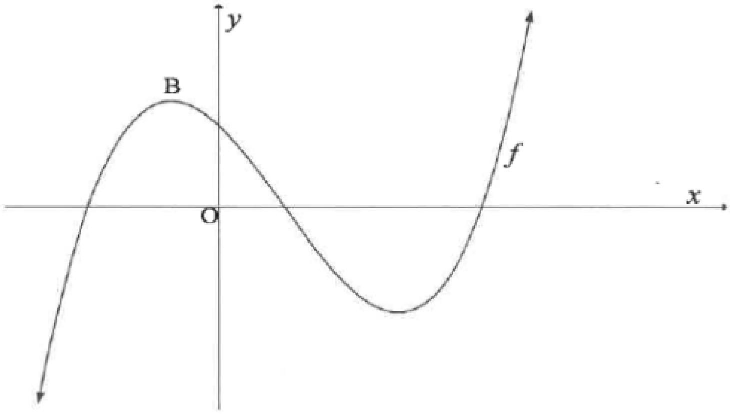
- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

draw a sketch graph of g indicating ALL relevant points. (3)

[15]

12.3 ACTIVITY 3

The sketch below represents the curve of $f(x) = x^3 + bx^2 + cx + d$. The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4.



3.1	Calculate the values of b, c and d.	(4)
3.2	Calculate the x-coordinate of B, the maximum turning point of f.	(4)
3.3	Determine an equation for the tangent to the graph of f at $x = -1$.	(4)
3.4	In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch.	(3)
3.5	For which value(s) of x is $f(x)$ concave upwards?	(2)
		[17]

12.4 ACTIVITY 4

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

4.1 Show that f has a point of inflection at $x = 2$. (5)

4.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

4.3 For which values of x will $y = -f(x)$ be concave down? (2)

4.4 Use your graph to answer the following questions:

4.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2)+3$. (2)

4.4.2 Claire claims that $f'(2) = 1$.

Do you agree with Claire? Justify your answer. (2)

[15]

12.5 ACTIVITY 5

Given: $f(x) = 2x^3 - 5x^2 + 4x$

5.1 Calculate the coordinates of the turning points of the graph of f. (5)

5.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

5.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

5.4 For which values of x will the graph of f be concave up? (3)

[14]

12.6 ACTIVITY 6

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

6.1 Calculate the x-coordinates of the stationary points of f. (3)

6.2 For which values of x is f concave down? (3)

6.3 Determine the values of x for which f is strictly increasing. (2)

6.4 If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5)

13 Practical Application

- Draw a sketch
- Draw up an equation, based on the problem, eg. f(x)
- Determine $f'(x)$
- Let $f'(x) = 0$, to deliver a maximum or minimum value for x
- Substitute x into f(x) to get the maximum or minimum value
- Remember:
    - $S(t) \rightarrow\ Distance$
    - $s'(t) \rightarrow instantaneous\ speed$
    - $s''(t) \rightarrow acceleration$

14 GUIDE TO MAXIMISE/MINIMISE AND RATE OF CHANGE

14.1 Maximise/Minimise

You might be given statement to formulate your own equation. Even if there are two variables

- Determine $f(x)\ and\ f'(x)$
- For MAXIMUM or MINIMUN it is where by $f'(x) = 0$
- Answer the question

14.2 Rate of change

Important hints or approach

- Rate → $is\ f'(x)$
- At Rest/ Stationary/ initial/ at first/Stop [t = 0 sec]
- Consider distance: $h(t) = at^3 + bt^2 + ct + d$ $(in\ metres)$
- Velocity: V =$h'(t)$ i.e rate of change in distance
- Acceleration= $v'(t)/h''(t)$ rate of change in velocity

DVA→ $D\big(f(x)\big) \rightarrow V\big(f'(x)\big) \rightarrow A\big(f''(x)\big)$

Maximum/ Minimum/ Fixed/ Largest

- $f'(x) = 0$
- Solve for x
- Substitute back to the original equation

14.3 COMMON ERRORS AND MISCONCEPTIONS

The vast majority of the learners do not attempt this questions because they are unable to derive the optimizing function from the given information.

14.4 SUGGESTIONS FOR IMPROVEMENT

(a) Learners should interrogate the optimum function even when it is given in a question. This will help their conceptual development.
(b) Teachers should ensure that there is enough time for learners to understand the application of calculus fully.
(c) Reading for understanding should be on going if learners are to improve their responses to word problems.

14.5 ACTIVITY 7

A closed rectangular box has to be constructed as follows:

- Dimensions: length (l), width (w) and height (h).
- The length (l) of the base has to be 3 times its width (w).
- The volume has to be $5\ \text{m}^3$.

The material for the top and the bottom parts costs R15 per square metre and the material for the sides costs R6 per square metre.

7.1 Show that the cost to construct the box can be calculated by: $\text{Cost} = 90w^2 + 48wh$ (4)

7.2 Determine the width of the box such that the cost to build the box is a minimum. (6)
[10]

14.6 ACTIVITY 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2+3t-6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

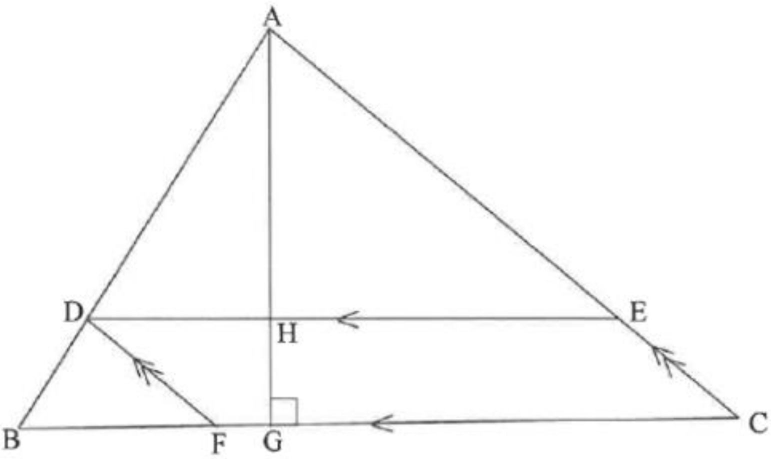
8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)
[8]

14.7 ACTIVITY 9

In ΔABC:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- BF : FC = 2 : 3.
- The perpendicular height AG is drawn intersecting DE at H.
- AG = t units.
- BC = $(5 - t)$ units.



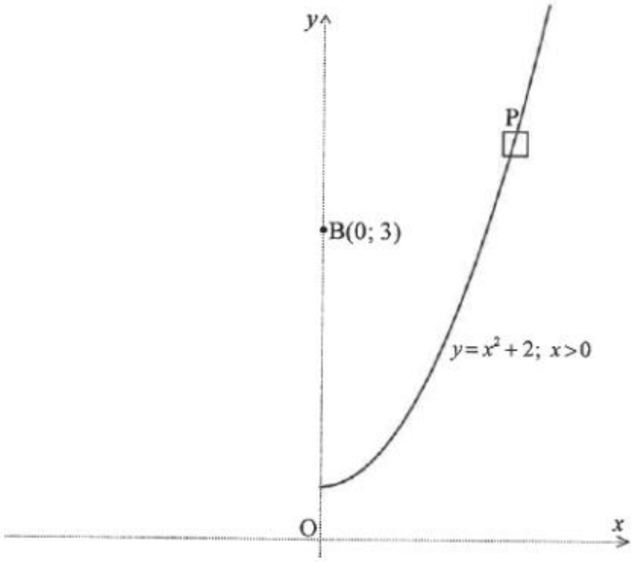
9.1 Write down AH : HG. (1)

9.2 Calculate t if the area of the parallelogram is a maximum.
(NOTE: Area of a parallelogram = base × $\perp$ height) (5)
[6]

14.8 ACTIVITY 10

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

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GRADE 12 PROBABILITY BOOKLET

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TOPIC: PROBABILITY

TERMINOLOGY AND DEFINATIONS

TERMINOLOGY	DEFINITIONS	
Probability	The likeness of an event occurring.	
Experiment	An activity to test what will happen if you do something.	
Outcome	A unique result of an experiment.	
Trials	The number of times an experiment is conducted.	
Sample space	A set of all possible outcomes of an experiment.	
Event	A subset of a sample space.	
CONCEPTS	HOW TO LEARN IT	RELEVANT FORMULAE AND KEYWORDS
Mutually exclusive events	Identify common items within the events. Cannot occur at the same time	$P(A\ or\ B) = P(A) + P(B)$ $P(A\ and\ B) = 0$
	Know the definition of mutually exclusive events using a formula.	Mutually exclusive $P(A\ and\ B) = 0$ ‘AND’ is the key to the concept
	Practice those examples that have the keywords ‘and’ and ‘or’	Check your answer after doing the calculations.
Addition rule	Elements of the addition rule are always the sample space	$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
	Know the identity	Not mutually exclusive $P(A\ and\ B) \neq 0$
	Write down the formula. Try using your own numbers for practicing correct substitution	Check your answers after doing your calculations
Complementary events	Add and get one	$P(A) + P(B) = 1$

		$P(A) + P(not\ A) = 1$ $P(A) = P(not\ B)$ $P(B) = P(not\ A)$
	Know the identity	"NOT" is the key to the concept
	Use the Venn diagrams and Tree diagrams to master Complimentary events.	Practice the exam-type questions.
Dependent and independent events	Dependent: Sample space get reduced.	Independent events
	Independent: Sample space stay the same.	$P(A\ and\ B) = P(A) \times P(B)$
	Product rule Correct substitution Replacing and not replacing	independent
	Use the tree diagram.	
The product rule	Keyword "and" means multiplication.	$P(A\ and\ B) = P(A) \times P(B)$
	Correct substitution	Independent "AND" is the key to the concept
	Practice using past papers and make sure that you see the keyword "AND" and substitute correctly.	
VENN diagrams	Representing listed information in a diagram.	No formula
	Start with numbers that affect all three events. Subtract the way out to those numbers that affect only two events. Consider each event in full.	VENN diagram

	Add to get the total of each circle. Add everything to get the sample space.	
	Make sure you read the question and identify the number of events. Draw your circles in a rectangular block to accommodate your Sample space.	
Tree diagrams	Understand that events must be presented in the form of branches.	
	Identify how many events do you have. Check what can happen i.e. probability.	With replacement Without replacement
	Make sure that your total outcomes are correct. When you add down the total outcomes, your answer must always give you one. When you go across the branches, you multiply the outcomes.	
Contingency tables	Frequencies of events represented on the table "AND" means intersection "OR" means union.	No formula unless you are proving **mutually exclusive** and **independen**t events.
	Totals (balancing the tables) Independent	Two things happening at the same time. Contingency table
	Make sure that your totals down and across always balances.	

Factorial notation and counting principles	Pattern in reverse order for consecutive numbers. Must be natural numbers. Arrangements and combinations.	$n!$ $m \times n$
	Know the formula: $\dfrac{n!}{(n-r)!}$	Arrangement In how many ways Put together
	Application of the formula is very important. Learn how to substitute the values correctly.	

CONTENT TO BE EXAMINED AS STIPULATED BY THE EXAMINATION GUIDELINES

1. Dependent events are examinable but conditional probabilities are not part of the syllabus (CAPS)
2. Dependent events in which an object is not replaced is examinable.
3. Questions that require the learner to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

MISCONCEPTIONS AND COMMON ERRORS

1. Inability to use the probability rules. For instance, the theory of $(at\ least\ one) = 1 - (A\ and\ B)$.
2. The inability to recognize the restriction of the probability when it lies between 0 and 1 such that $0 \leq P(A) \leq 1$
3. Learners are often incapable of representing given information on a tree diagram.
4. Learners lack the comprehension of the use of factorials to answer specific questions.

RECOMMENDATIONS

1. Teaching basic concepts cannot be overlooked. When learners understand the basic concepts, the easier it is for them to grasp complex concepts.
2. It must be emphasized that the probability of an event A lies in the interval $0 \leq P(A) \leq 1$
3. Reading for understanding must be a regular practice in the classroom. This should equip

learners with the skills to deal with word problems in assessment tasks.

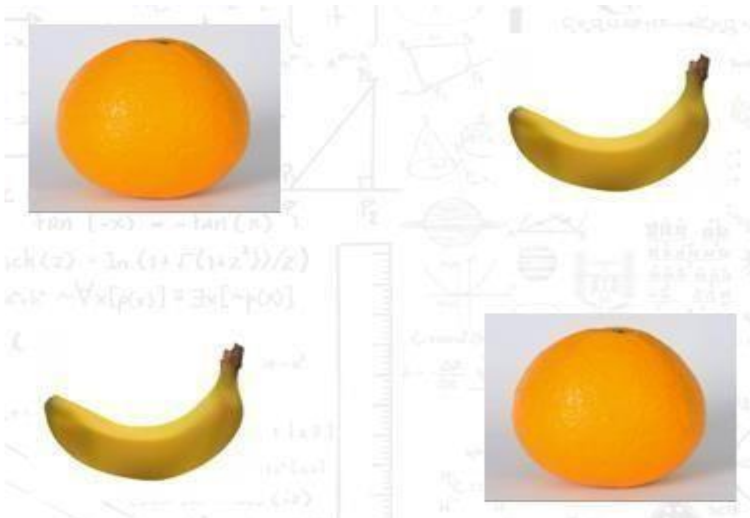
4. Teachers need to teach both tree diagrams and Venn diagrams thoroughly. These concepts should be examined in school-based assessment tasks throughout the FET phase.
5. Teach learners the fundamental counting principle in such a way that they will be able to base their answers on their reasoning, rather than on the rule.

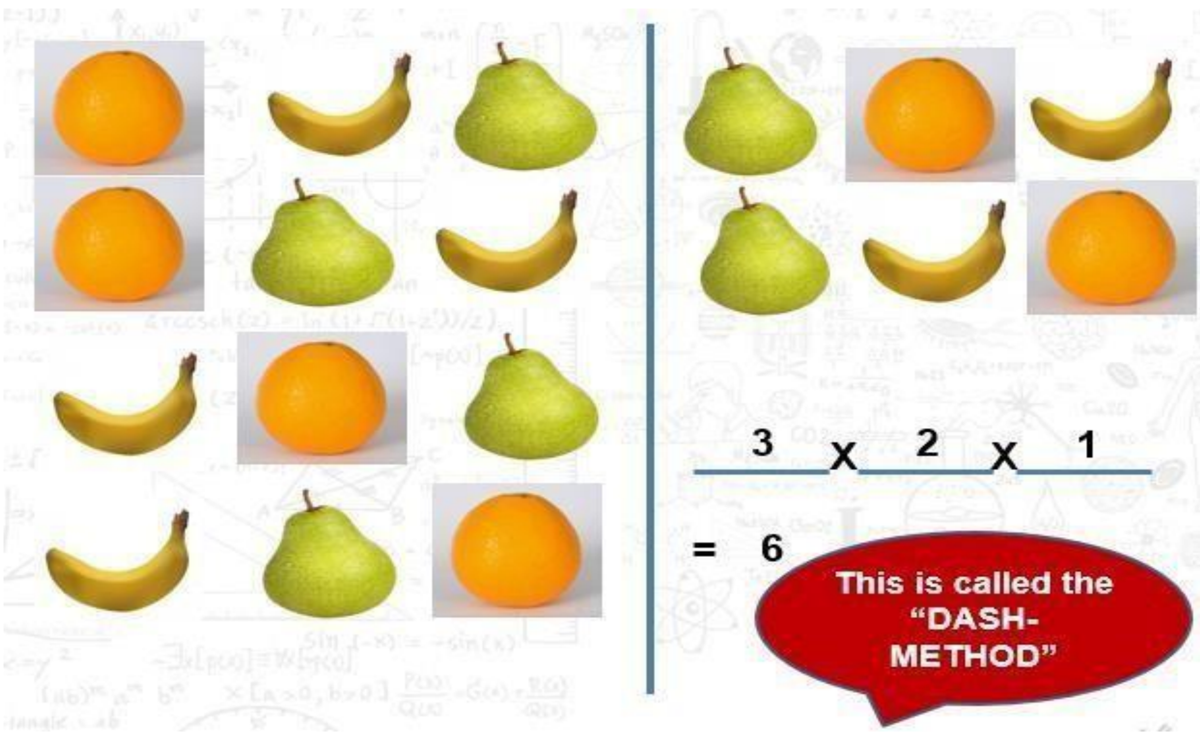
COUNTING PRINCIPLES:

A rule or a method used to calculate the total number of ways in which two or more items are arranged in a row.

How to arrange items in a row?

An item in a row is placed on either the left side or the right side of the other item.




3 x 2 x 1
= 6
This is called the "DASH-METHOD"

FACTORIAL NOTATION:

Using the DASH-method to arrange items in a row.

1. One item: $1 \times 1 = 1!$
2. Two items: $2 \times 1 = 2!$
3. Three items: $3 \times 2 \times 1 = 6 = 3!$
4. Four items: $4 \times 3 \times 2 \times 1 = 24 = 4!$

$$\therefore \boldsymbol{n}! = \boldsymbol{n} \times (\boldsymbol{n}-\mathbf{1}) \times (\boldsymbol{n}-\mathbf{2}) \times (\boldsymbol{n}-\mathbf{3}) \times \ldots \times 3 \times 2 \times \mathbf{1}$$

This is called the "factorial notation"

Note: in cases where repetition is allowed, we can use:

$$number\ of\ possibilities = n^r$$

Where n = the number of options to choose from

r = the number of times we choose from these options.

FIXED POSITIONS AND GROUPING OF ITEMS:

Example 1

A banana Airways aeroplane has 6 seats in each row.

1.1. How many possible arrangements are there for 6 people to sit in a row of 6 seats?

$\mathbf{6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720}$

1.2. Xoliswa, Anele and four other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anele must sit next to each other?

$$\mathbf{5! \times 2! = 240}$$

1.3. Mary and five other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row?

$$\mathbf{5 \times 4 \times 3 \times 2 \times 1 \times \frac{1}{\boldsymbol{Mary}} = 120}$$

This seat is fixed. Only Mary will occupy it

$$\boldsymbol{P} = \mathbf{\frac{120}{720} = \frac{1}{6}}$$

EXAMPLE 2

There are 7 different shirts and 4 different pairs of trousers in a cupboard. The clothes have to be hung on the rail.

2.1. In how many different ways can the clothes be arranged on the rail?

$$11! = 39\ 916\ 800$$

2.2. In how many different ways can the clothes be arranged if all the shirts are to be hung next to each other and the pairs of the trousers are to be hung next to each other on the rail?

$$\frac{2!}{groups} \times \frac{4!}{pairs\ of trousers} \times \frac{7!}{shirts} = 241\ 920$$

2.3. In how many ways can a pair of trousers hang at the beginning of the rail and a shirt will hang at the end of the rail?

$$\frac{\mathbf{4}}{\textbf{one pair of trousers at the beginning}} \times \frac{\mathbf{9!}}{\textbf{remaining clothes to be hung}} \times \frac{\mathbf{7}}{\textbf{one shirt at the end}} = 10\ 160\ 640$$

2.4. What is the probability that a pair of trousers will hang at the beginning of the rail will hang at the end of the rail?

$$P = \frac{10\ 160\ 640}{39\ 916\ 800} \Longrightarrow P = \frac{14}{55}$$

EXAMPLE 3

The digits 0,1,2,3,4,5,6, and 7 are used to make four-digit codes.

3.1 How many unique codes are possible if the digits can be repeated?

$$8 \times 8 \times 8 \times 8 = 4096$$

3.2 How many unique codes are possible if the digits cannot be repeated?

$$8 \times 7 \times 6 \times 5 = 1680$$

3.3 In the case where digits may be repeated, how many codes are numbers that are greater than 2000 and even?

$$6 \times 8 \times 8 \times 4 - 1 = 1535$$

Example 4

Four-digit codes (not beginning with 0) are to be constructed from the set of digits: {1,2,3,4,5,6,7,8,0}

4.1. How many four-digit codes can be constructed, if repetition of digits is not allowed?

$$8 \times 9 \times 9 \times 9 = 5832$$

4.2. How many four-digit codes can be constructed, if repetition of digits is not allowed?

$$8 \times 8 \times 7 \times 6 = 2688$$

4.3. Calculate the probability of randomly constructing a four-digit code which is divisible by 5 if repetition of digits is allowed.

$$8 \times 9 \times 9 \times 2 = 1296$$

IDENTICAL ITEMS IN A ROW:

Example 5

Consider the word WINNERS.

5.1. How many word arrangements can be made with this word if the repeated letters are **treated as different** letters?

$$7! = 5040$$

5.2. How many word arrangements can be made with this word if repeated letters are treated **as identical**?

$$\frac{7!}{2!} = 2520$$

5.3. How many word arrangements can be made with this word if the word if the starts and ends with the same letter?

$$\frac{1}{N} \times 5! \times \frac{1}{N} = 120$$

5.4. How many word arrangements can be made with this word if the word starts with W and ends with the S?

$$\frac{\frac{1}{W} \times 5! \times \frac{1}{S}}{2!} = \frac{5!}{2!} = 60$$

Example 6

Consider the word ADVERTISEMENT.

6.1. How many word arrangements can be made with this word if the repeated letters are treated as different letters?

$$13! = 6\ 227\ 020\ 800$$

6.2. How many word arrangements can be made with this word if the repeated letters are treated as identical?

$$\frac{13!}{3! \times 2!} = 518\ 918\ 400$$

6.3. How many word arrangements can be made with this word if the word starts and ends with the same letter?

If it starts with "E".

$$1 \times \frac{11!}{3!} \times 1 = 6\ 652\ 800$$

If it starts with "T"

$$1 \times \frac{11!}{2!} \times 1 = 19\ 958\ 400$$

$$Total = 26\ 611\ 200$$

6.4. What is the probability that the word starts and ends with the same letter? Note: $probability = \frac{Number\ of\ possibilities\ meeting\ the\ condition}{total\ number\ of\ possibilities}$

$$P = \frac{26\ 611\ 200}{518\ 918\ 400}$$

Then $P = \frac{2}{39}$

ACTIVITIES FOR COGNITIVE LEVEL 1 AND LEVEL 2

Given:

- $\text{P(A)} = 0{,}12$
- $\text{P(B)} = 0{,}35$
- $\text{P(A or B)} = 0{,}428$

Determine whether events A and B are independent or not. Show ALL relevant calculations used in determining the answer. (4)

QUESTION 2

For the two events A and B, it is given that:

- $\text{P(A)} = 0{,}2$
- $\text{P(B)} = 0{,}63$
- $\text{P(A and B)} = 0{,}126$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

QUESTION 3

The events A and B are independent.

P(A) = 0,4 and P(B) = 0,5

Determine:

1. P (A and B) (2)
2. P (A or B) (2)
3. P (not A and not B) (3)

QUESTION 4

Events A and B are independent of one another, with $(A) = 0{,}4$ and $P(A\ and\ B) = 0{,}12$.
Determine:

1. P(B) (2)
2. P (A or B) (2)
3. P (not A and not B) (3)

QUESTION 5

Events A and B are mutually exclusive. It is given that:

- $\mathrm{P(B) = 2P(A)}$
- $\mathrm{P(A\ or\ B) = 0{,}57}$

Calculate P(B). (3)

QUESTION 6

The probability of events A and B occurring are denoted by P (A) and P (B) respectively.

For any two events A and B it is given that:

- $P(B') = 0{,}28$
- $P(B) = 3P(A)$
- $P(A\ or\ B) = 0{,}96$

Are the events A and B mutually exclusive? Justify your answer. (4)

QUESTION 7

A and B are two events in a sample space.

$\mathrm{P\ (not\ A) = 0{,}45\ and\ P(B) = 0{,}35}$

1. Determine P(A) (1)
2. Determine P (A or B) if A and B are mutually exclusive events. (2)
3. Determine P (A and B) if A and B are independent events. (2)

QUESTION 8

if $P(A)=\frac{3}{8}$ *and* $P(B)=\frac{1}{4}$, find:

1. P (A or B) if A and B are mutually exclusive events. (1)

QUESTION 9

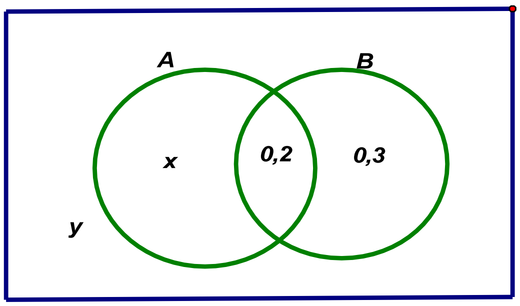
Given: $\mathrm{P\ (A) = 0{,}2\ and\ P\ (B) = 0{,}7}$. Calculate $\mathrm{P(A\ or\ B)}$ if:

1. A and B are mutually exclusive (2)

2. A and B are independent (3)
3. Represent the information on a Venn diagram. (3)
4. Calculate P(A). (3)

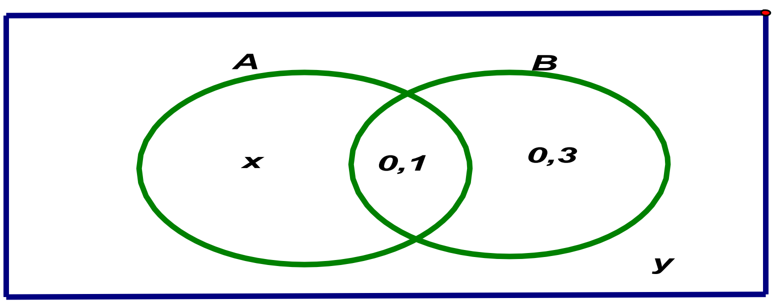
QUESTION 10

If A and B are independent events, find the values of x and y. Show all the workings. (4)



QUESTION 11

A and B are independent events.



Determine the values of x and y. all calculations must be shown. (5

QUESTION 12

Zebra High School offers only two sporting activities, namely rugby and hockey.
The following information is given:

- There are 600 learners in the school.
- 372 learners play hockey.
- 288 learners play rugby.
- 56 of the learners play NO sport.

The number of learners that play both hockey and rugby is x

1. Represent the given information in a Venn diagram, in terms of x. (3)
2. Calculate the value of x. (2)
3. Are the events playing rugby and playing hockey mutually exclusive? Justify your answer. (3)

QUESTION 13

At a school for boys there are 240 learners in Grade 12. The following information was gathered about participation in school sport.

- 122 boys play rugby (R)
- 58 boys play basketball (B)
- 96 boys play cricket (C)
- 16 boys play all three sports.
- 22 boys play rugby and basketball.
- 26 boys play cricket and basketball.
- 26 boys do not play any of these sports.

Let the number of learners who play rugby and cricket only be x.

1. Draw a Venn diagram to represent the above information
2. Determine the number of boys who play rugby and cricket.
3. Determine the probability that a learner in Grade 12 selected at random: (leave your answer correct to
4. THREE decimal places)
   4.1. Only plays basketball (2)
   4.2. Does not play cricket (2)
   4.3. Participates in at least two of these sports. (2)

QUESTION 14

A survey of 80 students at a local library indicated the reading preferences below:

44 read the National Geographic magazine.

33 read the Getaway magazine.

39 read the leadership magazine.

23 read both National Geographic and Leadership magazine.

19 read both Getaway and Leadership magazines.

9 read all three magazines.

69 read at least one magazine.

1. How many students did not read any magazine? (1)
2. Let the number of students who read National Geographic and Getaway, but not Leadership, be represented by x. Draw a Venn diagram to represent reading preferences. (5)
3. Hence show that $x = 5$. (3)

QUESTION 15

Complaints about a restaurant fell into three main categories: the menu (M), the food (F) and the service (S). In total 173 complaints were received in a certain month. The complaints were as follows:

- 110 complained about the menu.
- 55 complained about the food.
- 67 complained about the service.
- 20 complained about the menu and the food, but not the service.
- 11 complained about the menu and the service, but not the food.
- 16 complained about the food and the service, but not the menu.
- The number who complained about all three is unknown.

1. Draw a Venn diagram to illustrate the above information. (6)
2. Determine the number of people who complained about ALL THREE categories. (3)

QUESTION 16

A school organized a camp for their 103 Grade 12 learners. The learners were asked to indicate their food preferences for the camp. They had to choose from chicken, vegetables, and fish.
The following information was collected:

- 2 learners do not eat chicken, fish or vegetables.
- 5 learners eat only vegetables.
- 2 learners only eat chicken.
- 21 learners do not eat fish.
- 3 learners eat only fish.
- 66 learners eat chicken and fish.
- 75 learners eat vegetables and fish.

Let the number of learners who eat chicken, vegetable and fish be x.

1. Draw an appropriate Venn diagram to represent the information. (7)
2. Calculate the value of x. (2)
3. Calculate the probability that a learner, chosen at random:
   3.1. Eats only chicken and fish, and no vegetables. (2)
   3.2. Eats any TWO of the given food choices: chicken, vegetables and fish. (2)

QUESTION 17

A survey of 2 140 teachers revealed that certain learners experience problems that negatively affect their learning.
The following data on the various problems was obtained:

- 890 teachers said that learning was negatively affected by children being abused (A).
- 680 teachers said that learning was negatively affected by malnutrition (N).
- 120 teachers said that learning was negatively affected by a lack of parental support (P) and children being abused (A).
- 110 teachers said that learning was negatively affected by a lack of parental support (P) and malnutrition (N).
- 140 teachers said that learning was negatively affected by children being abused (A) and malnutrition (N).
- An unknown number of teachers (x) said that learning was negatively affected only by a lack of parental support (P).
- Every teacher said that learning was negatively affected by at least one problem.

1. Draw a Venn diagram to represent the above situation. (6)
2. Calculate the number of teachers who said that a lack of parental support was a problem. (3)
3. Calculate the probability that a teacher selected at random from this group said that learners had exactly two problems. (3)

QUESTION 18

In a survey of 120 people, it was found that: 50 read Mirror, 52 read Sowetan, 52 read Daily Sun, 18 read both Mirror and Daily Sun, 22 read both Mirror and Sowetan, 16 read both Sowetan and Daily Sun, 6 read all three newspapers.

1. Draw a Venn diagram to represent this information (4)
2. What is the probability that a randomly chosen person read exactly one newspaper? (2)
3. Calculate the probability of selecting a person who does not read any of these newspapers. (2)

CONTINGENCY TABLES

QUESTION 1

The table below represents the favorite sport for 120 learners.

	Boys	Girls	**Total**
Golf	32	12	**44**
Tennis	18	23	**41**
Squash	20	15	**35**
Total	**70**	**50**	**120**

Determine the probability that (if a learner is selected at random):

1. The learner who is selected, is a boy who plays tennis or squash. (2)
2. A learner who plays tennis is selected. (1)
3. A learner who plays tennis is selected. (1)
4. Are the events choosing golf as a favorite sport and being a boy, independent?
   Show ALL calculations to support your answer. (4)

QUESTION 2

A survey was conducted asking 60 people with which hand they write and what color hair they have. The results are summarized in the table below:

		HAND USED TO WRITE WITH		
		Right	Left	Total
HAIR COLOUR	Light	**a**	**B**	20
	Dark	**c**	**D**	40
	Total	48	12	60

The survey concluded that the 'hand used for writing' and 'hair color' are independent events. Calculate the values of a, b, and c. (5)

QUESTION 3

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from, were to:

- Go to the coast.
- Visit a game park.
- Stay at home.

The results were recorded in the table below:

	Coast	**Game Park**	**Home**	**Total**
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

Determine the probability that a randomly selected staff member:

1. Is male (1)
2. Does not prefer visiting a game park (2)
3. Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)

QUESTION 4

Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit. The results are shown in the table below:

	Male	Female	Total
Tea	20	40	60
Coffee	***b***	***c***	80
Fruit juice	***d***	***e***	20
Total	60	100	***a***

1. Write down the value of a. (1)
2. What is the probability that a randomly selected passenger is male? (2)
3. Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of b (4)

ACTIVITIES FOR COGNITIVE LEVEL 3 AND 4

QUESTION 1

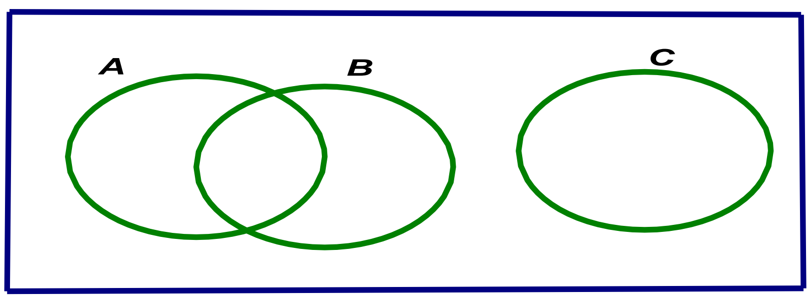
For a sample space S and events A and B, it is given that:

- $P([not\ A]\ and\ B) = P(A'\ and\ B) = \frac{5}{12}$
- $P(A\ and\ B) = \frac{1}{6}$
- $P(1 - [A\ or\ B]) = \frac{1}{3}$

Represent this information on the Venn diagram. (3)

QUESTION 2

Consider events A, B and C represented in the Venn diagram below. Events A and B are independent.



It is given that P(A) = 0.45; P(B) =0.3 and P(C) =0.32

1. Mary claims that events A and B are mutually exclusive. Explain why you agree or disagree with Mary. (2)

Calculate the probability that, of the three events;

1. At least one of B or C occurs. (2)
2. At least one of A or B occurs. (2)

QUESTION 3

A survey of 2 140 teachers revealed that certain learners experience problems that negatively affect their learning.

The following data on the various problems was obtained:

- 890 teachers said that learning was negatively affected by children being **abused** (A).
- 680 teachers said that learning was negatively affected by **malnutrition** (N).

- 120 teachers said that learning was negatively affected by a **lack of parental support** (P) and children being **abused** (A).
- 110 teachers said that learning was negatively affected by a **lack of parental support** (P) and **malnutrition** (N).
- 140 teachers said that learning was negatively affected by children being **abused** (A) and **malnutrition** (N).
- An unknown number of teachers (x) said that learning was negatively affected only by a **lack of parental support (P).**
- Every teacher said that learning was negatively affected by at least one problem.

1. Draw a Venn diagram to represent the above situation. (6)
2. Calculate the number of teachers who said that a lack of parental support was a problem.(3)

QUESTION 4

Lebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and a 30% chance of passing both the Music and Art examination.

1. Calculate the probability that Lebo will pass the Music or Art examination. (3)

TREE DIAGRAMS:

QUESTION 1

The probability that it will rain on a given day is 63%. A child has a 12% chance of falling in dry weather and is three times as likely to fall in wet weather.

1.1 Draw a tree diagram to represent all outcomes of the above information. (6)

1.2 What is the probability that a child will not fall on any given day? (3)

1.3 What is the probability that a child will fall in dry weather? (2)

QUESTION 2

1.1 During summer in a certain city in South Africa the probability of a sunny day is $\frac{4}{7}$ and the probability of a rainy day is $\frac{3}{7}$.

- If it is a sunny day, then the probability that Vusi cycles to work is $\frac{7}{10}$, the probability that Vusi drives to work is $\frac{1}{5}$ and the probability that Vusi takes the train to work is $\frac{1}{10}$

- If it is a rainy day, then the probability that Vusi cycles to work is $\frac{1}{9}$, the probability that.

Vusi drives to work is $\frac{5}{9}$ and the probability that Vusi takes the train to work is $\frac{1}{3}$.

1.1.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes. (5)

1.2 For a day selected at random, what is the probability that:

1.2.1 It is rainy and Vusi will cycle to work. (2)

1.2.2 Vusi takes the train to work. (3)

1.2.3 If Vusi works 245 days in a year, on approximately how many occasions does he drive to work? (4)

QUESTION 3

A box contains 8 red marbles and 10 blue marbles. Two marbles are selected randomly, one after the other, without replacement.

Draw a tree diagram to represent all outcomes of the above information. (4)

- Determine the probability that:

Both marbles are red. (2)

At least one marble is blue. (1)

A red and a blue marble were selected in that order. (2)

QUESTION 4

In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy. At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at a school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at a school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6)

QUESTION 5

There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

5.1 Calculate the probability that the first learner chosen is a boy. (1)

5.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate ALL possible outcomes on your diagram (9)

5.3 Calculate the probability that a boy, then a girl and then another boy is chosen in that order. (3)

5.4 Calculate the probability that all three learners chosen are girls. (2)

5.5 Calculate the probability that at least one of the learners chosen is a boy. (3)

QUESTION 6

A bag contains 6 red balls, 8 green balls and unknown number of yellow balls. The probability of randomly choosing a green ball from the bag is 25%.

6.1 Show that there are 32 balls in the bag. (5)

6.2 A ball is drawn from the bag; the colour is recorded and is not returned to the bag. Thereafter another ball is drawn from the bag, the colour is recorded and it is also not returned to the bag.

Draw a tree diagram to represent ALL the possible ways in which the two balls could have been drawn from the bag. Show the probabilities associated with EACH branch as well as the outcomes. (6)

6.3 Calculate the probability that 2 balls drawn from the bag will have the same color. (3)

QUESTION 7

Alfred and Barry have an equal chance of winning a point in a game.

7.1 Draw a tree diagram to represent the situation after a total of 3 points have been contested. Indicate on your diagram the probabilities and all the outcomes associated with each branch. (5)

7.2 Calculate the probability that Barry would have won all 3 points. (2)

7.3 Calculate the probability that Alfred would have won 2 points and Barry would have won 1 point of the 3 points contested. (2)

QUESTION 8

There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same color from the bag is 52%

Calculate how many orange balls are in the bag. (6)

QUESTION 9

A bag contains 12 blue balls, 10 red balls and 18 green balls. 2 balls are chosen at random without replacement.

Determine the probability:

9.1 If the two balls chosen at random are green. (3)

9.2 If the two balls chosen at random are blue and red. (3)

QUESTION 10

The success rate of the Fana soccer team depends on a number of factors. The fitness of the players is one of the factors that influence the outcome of a match.

- The probability that all the players are fit for the next match is 70%.
- If all the players are fit to play the next match, the probability of winning the next match is 85%.
- If there are players that are not fit to play the next match, the probability of winning the match is 55%.

Based on the fitness alone, calculate the probability that the Fana soccer team will win the next match. (5)

QUESTION 1

The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters.

How many different arrangements are possible if:

1. Letters may be repeated (2)
2. Letters may not be repeated (2)
3. The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed. (4)

QUESTION 2

Consider the letters **OMOLEMO.**

1. In how many different unique combinations can these letters be arranged? (3)

2. If the letter M must be placed first, how many different unique arrangements will there now be? (2)

QUESTION 3

The digits from 1 to 9 are used to make 5-digit codes.

1. Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition. (2)
2. Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated. (3)
3. Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digit is allowed. (2)

QUESTION 4 [5]

The nine letters of the word 'EQUATIONS' are used to form different five-letter codes.

1. How many different five-letter codes can be formed from the nine different letters in the word 'EQUATIONS'? (2)
2. How many different five-letter codes can be formed from the letters in the word 'EQUATIONS' by using all the consonants and one vowel? (3)

QUESTION 5 [8]

Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.

5.1 How many personal identity numbers (PINs) can be made if

I. Digits can be repeated. (2)

II. Digits cannot be repeated. (2)

5.2 Suppose that a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4)

QUESTION 6

Five South Africans, three British citizens, four Germans and two Americans arrive for a congress.

1. In how many ways can they be arranged if they sit next to each other in a row (1)
2. In how many ways can they be arranged so that those of the same nationality sit together in a row? (2)

Consider the letters of the word: **PROBABILITY.**

1. How many unique word arrangements can be made from this word if the repeated letters are treated as identical? (3)
2. How much unique word arrangement can be made if the word ends with the letter R? (2)

QUESTION 7

A South African music group plan a concert tour with performances in Durban, East-London, Port Elizabeth, Cape Town, Bloemfontein, Johannesburg, and Polokwane. In how many different ways can their tour be planned if:

1. There are no limits. (2)
2. The first performance must be in Cape Town and the last one in Polokwane. (2)
3. The performances in the four coastal cities (cities next to the sea) must be done one after another. (3)
4. The concerts take place in only any 4 of the 7 cities. (2)

QUESTION 8

A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

1. All the letters of the alphabet can be used. (2)
2. The password must start with a 'D' and end with an 'L' (2)

Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

1. In how many ways can ALL the cars be parked? (2)
2. The three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)

QUESTION 9

Consider the word **MATHS.**

1. How many different 5-letter arrangements can be made using all the above letters? (2)
2. How many different 5-letter arrangements can be made using all the above letters? (2)
3. Determine the probability that the letters S and T will be the first two letters of the arrangements in the QUESTION ABOVE. (3)

QUESTION 10

5 girls and 3 boys are to sit on a bench in a straight line.

1. In how many ways can they all sit on the bench? (3)
2. In how many ways can they all sit on the bench with girls seated together? (2)
3. Determine the probability that all girls are seated next to each other. (2)

QUESTION 11

Three boys and four girls go to the cinema. They are seated together in one row as a group.

1. Calculate the total number of possible ways of arranging the whole group. (2)
2. Determine the total number of ways of arranging the group if the girls want to be seated together. (3)
3. Calculate the probability that the girls are seated together.
4. Determine the probability that they will all be seated girl, boy, girl, boy etc. until all seven are seated. (3)

QUESTION 12

Eight boys and seven girls are to be seated randomly in a row. What is the probability that:

A. The row has a girl at each end. (3)
B. The row has girls and boys sitting in alternate positions? (3)

QUESTION 13

A Banana Airways aeroplane has 6 seats in each row.

1. How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)
2. Vincent, Nathaniel and four other passengers sit in a certain row on a banana Airways flight. In how many different ways can these 6 passengers be seated if Vincent and Nathaniel must sit next to each other? (2)
3. Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? (4)

QUESTION 14

Dendron Primary has a sports awards ceremony. The school has a basketball team consisting of 5 players and volleyball team consisting of 6 players.

1. All the basketball players sit in a single row at the ceremony. There are no restrictions on who sits in which position. In how many ways can they be seated?
2. The decision is taken that the captain must sit in the first seat of the row. The two vice-captains have to be seated next to each other in any of the remaining seats. In how many ways can the basketball players be seated now? (3)

3. After the interval, the basketball team and the volleyball team sit in the same row at the ceremony. Calculate the probability that the basketball players will sit together, and the volleyball players will sit together.
   Assume that seating positions are allocated randomly. Give your answer as simplified fraction. (3)

QUESTION 14

The Eastern Cape requires new codes for number plates. The new codes consist of four letters followed by four digits, as shown below. All codes end with EC.

BCDF 3856 EC

The vowels (A, E, I, O, U) and Q may not be used and digits 1 to 9 are used. Letters and digits may be repeated.

1. Determine how many number plates with different codes can be made. (3)

2. Determine the probability that a code that is randomly selected will consist of even digits which are not the same. (2)

QUESTION 15

There are 7 different shirts and 4 different pairs of trousers in a cupboard. The clothes have to be hung on the rail.

1. In how many ways can the clothes be arranged on the rail? (2)
2. In how many ways can the clothes be arranged if all the shirts are to be hung next to each other and the pairs of trousers are to be hung next to each other on the rail. (3)
3. What is the probability that a pair of trousers will hang at the beginning of the rail and a shirt will hang at the end of the rail? (4)
4. What is the probability that a pair of trousers will hang at the beginning of the rail and a shirt will hang at the end of the rail? (4)

QUESTION 16

Koketso and Marvin are to sit with five other friends in a church in one row. There are seven empty chairs in that row.

1. In how many different ways can they all sit together? (2)
2. In how many different ways can they sit if Koketso and Marvin should not sit next to each other? (4)
3. Koketso first sat on a chair at one end of the row. He now decides to change his initial position. What is the probability that he will sit on a chair at the other end of the row?